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D3 brane action and fermion zero modes in presence of background flux

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Abstract: We derive the fermion bilinear terms in the world volume action for a $D3$ brane in the presence of background flux. In six-dimensional compactifications non-perturbative corrections to the superpotential can arise from an euclidean D3-brane instanton wrapping a divisor in the internal space. The bilinear terms give rise to fermion masses and are important in determining these corrections. We find that the three-form flux generically breaks a U(1) subgroup of the structure group of the normal bundle of the divisor. In an example of compactification on T^6/Z_2, twelve of the sixteen zero modes originally present are lifted by the flux.

Keywords: D-branes, Superstring Vacua.

Contents

1. Introduction

Flux compactifications have attracted considerable attention recently. They are of interest from the point of view of string cosmology, phenomenology, and the general study of string theory vacua with $\mathcal{N} \leq 1$ supersymmetry.

Much more needs to be done to understand these compactifications better. In particular it should be possible to understand the full superpotential, including non-perturbative corrections, for these compactifications in greater depth. The superpotential has already proved amazingly useful in the study of supersymmetric string theories and field theories. And we can hope that its study for flux compactifications will prove similarly rewarding.

An immediate motivation for our work is to understand KKLT [1] type compactifications better. These compactifications were first formulated in the context of IIB string theory of Calabi-Yau orientifolds or related F-theory compactifications. Here the non-perturbative corrections to the superpotential play a vital role in stabilising the Kahler moduli [2].

The study of non-perturbative corrections to the superpotential -in the closely related context of M-theory on a Calabi Yau fourfold - was pioneered by Witten [3]. He showed that non-perturbative corrections due to euclidean 5-branes wrapping divisors in the four-fold could arise if the divisor satisfied a particular topological criterion, namely its arithmetic genus was unity. In Witten's analysis it was assumed that a particular U(1) symmetry, which is a subgroup of the structure group of the normal bundle to the divisor, was unbroken. The arithmetic genus is an index which counts the number of zero modes after grading by this symmetry. More recently, in [4], a class of non-perturbative corrections were studied in a IIB compactification on $K3 \times T^2/Z_2$, This is related to M theory on $K3 \times K3$. Evidence was found that in the presence of flux the U(1) symmetry mentioned above is broken. And it was argued that as a result non-perturbative corrections could arise even in situations where the arithmetic genus in not unity.

In this paper we consider a euclidean D3 brane wrapping a 4-cycle in a non-trivial background including flux. Using the method of gauge completion we calculate all the terms in the action of the D3 brane which are bilinear in fermions. These terms explicitly show that the U(1) symmetry of rotations normal to the 4-cycle is indeed broken in the presence of flux. As a result the zero mode counting will be altered in general and modes with the same U(1) charge can pair up and get heavy. In a particular example of IIB on an T^6/Z_2 orientifold we examine the resulting fermion zero modes. A non-perturbative correction to the superpotential requires two zero modes. Ignoring flux, there are sixteen zero modes. Including flux, we find for a large class of divisors that twelve of the sixteen zero modes are lifted. Although this still leaves four zero modes, which is too many for a correction to be possible, the example illustrates the "efficacy" of flux in lifting zero-modes.

This paper is only a first step towards a more complete understanding. One would like to use the results obtained here to understand the number of zero modes which arise more generally. And when the zero mode counting allows for a correction to the superpotential, calculate these corrections. These are interesting questions which we leave for the future.

We should also comment on some of the other relevant literature here. We use the method of gauge completion to determine the world volume theory for the D3 branes. This method is clearly discussed in the paper by [5, 6]. Our analysis very closely parallels the work by Grana [7]. The only difference is that we are interested in the more general situation where the D3 brane is not necessarily transverse to the compactified directions. Our results are in agreement after T-duality with those obtained for the D0 brane by [8]. This is a useful check on our work. The fermion bilinear terms for a Dp brane in a general background have in fact been obtained earlier in the significant papers by Marolf, Martucci and Silva, [10, 11]. Our results can be obtained as a gauge fixed version of their's for the D3-brane case and agree. This constitues an important check on our results and methods. Finally, while we were working on this project, the paper by Kallosh and Sorokin [9]

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appeared which determined the fermion bilinear terms for an M 5-brane. Using duality this can be related to the action we calculate here. After identifying the relevant gauge conditions etc we have found substantial agreement.[1]

This paper is planned as follows. The method of gauge completion, which we use to deduce the fermion bilinear terms, is first briefly explained in section 2. Following that we illustrate its use for some examples and then present the main results determining the superfields in IIB theory in terms of the component supergravity fields upto the required order. In section 3 we discuss the resulting D3 brane action. Our results are checked against those for a D0 brane using T-duality in section 4, we also comment on the agreement with other resulsts in the literature. In section 5 we discuss an example of a compactification on a T^6/Z_2 orientifold and calculate the resulting zero modes for a class of divisors. Last, but not least, are the six appendices which contain some of the important detailed calculations of the paper.

2. Gauge completion

The approach we will follow for constructing the world volume action of the D3 brane is straightforward. Given a IIB background in superspace the D3-brane action can be constructed by appropriately pulling back the background fields on to the brane world volume, as is explained in [12 − 16]. This action has the required supersymmetry and is also κ symmetric, for on-shell backgrounds. We are interested here in the D3-brane action in terms of the standard component fields of IIB supergravity. So we will first express the superfields of IIB theory in terms of the component supergravity fields by a process called "gauge completion". Once this is done we use the construction mentioned above in terms of the superfields to obtain the required action.

The method of gauge completion is discussed in [5]. It was applied to the supermembrane in [6]. Our work will closely parallel the paper of Grana who used an identical strategy. The only difference is that [7] was interested in the case where the D3 brane is transverse to the compactified directions. We will be interested in obtaining the more general answer. Our primary interest is in applying these results to the case of a euclidean D3 brane which wraps a four cycle in the internal directions. In this section and the next two, where we construct the world volume action and compare our results with those obtained in the D0 brane case respectively, we will work in Minkowski space. The required transformations to go to euclidean space will be discussed in section 5 before we apply our results in an explicit example.

The idea behind gauge completion is to expand the superfields in terms of the fermionic coordinates, θ, and express each term in the expansion in terms of the component fields of supergravity. By the component fields here we mean the fields which appear in the usual discussion of IIB supergravity, for example, section 13, [17] and, [18]. For an on-shell background these satisfy the equations of motion of the IIB theory. To lowest order in the θ expansion of the superfields the component supergravity fields which appear are known.

[1]We thank R. Kallosh and D. Sorokin for discussions in this regard.

To go to higher orders one follows an iterative procedure. The idea is that superfields must transform as appropriate tensors under general bosonic and fermionic coordinate transformations in superspace. In particular this included supersymmetry transformations which are translations in the fermionic coordinates. Since the supersymmetry transformations for the component supergravity fields are known this allows us to express the higher order terms in the θ expansion in terms of the lower order ones. Obtaining an answer to all orders for a general background in this way is computationally quite non-trivial. Luckily, since we are only interested in terms which are bilinear in the fermions here, it will suffice to carry out this expansion upto second order in θ at most.

In this section we will first illustrate this procedure for the dilaton superfield, $\hat{\phi}$ and the NS-NS two-form superfield, $\hat{B}_{MN}$ and then present the results for the other superfields towards the end. The calculations are somewhat cumbersome and several details are presented in the appendix.

Let us begin by explaining our notation. We denote superspace coordinates by $Z^M = (x^m, \theta^\mu)$, which stand for the bosonic and fermionic components respectively. The indices $\{M, N, \ldots\} = \{m, n, \ldots, \mu, \nu, \ldots\}$ denote curved (super) coordinates where (m, n) denote Bosonic indices and (μ, ν) fermionic indices. Tangent space indices are given by $\{A, B, \ldots\} = \{a, b, \ldots, \alpha, \beta, \ldots\}$, with (a, b) denoting bosonic and (α, β) fermionic indices. We will use real 16 component Majorana-Weyl spinors, our conventions for the Gamma matrices are summarised in appendix A. The spinor indices α, β should be viewed as composite indices standing for the tensor product of a Majorana-Weyl index and an additional SO(2) index.

Our notation for the superfields is as follows. A generic superfield is represented by $\hat{F}_{MN\cdots}$ (with a ‘ ˆ ’ over the field). The dilaton superfield, whose lowest component is the dilaton, ϕ, is denoted by $\hat{\phi}$, the vierbein superfield by $\hat{e}^A_M$ and similarly for $\hat{B}_{MN}, \hat{C}, \hat{C}_{MN}, \hat{C}_{MNPQ}$ which denote the superfields containing the NS-NS two form, and the RR zero, two and four forms respectively.

Our conventions in superspace are the same as those in [19]. Derivatives with respect to θ are left derivative. Superspace differentials satisfy the property that $dZ^M \wedge dZ^N = (-1)^{(1+MN)} dZ^N \wedge dZ^M$, where $MN = +1$ when both M, N are fermionic, and zero otherwise. A differential two-form for example is given in terms of components by $\hat{B} = dZ^N dZ^M \hat{B}_{MN}$, and so on.

Under a superspace diffeomorphism $Z^M \to Z^M + \Sigma^M(Z)$, the dilaton superfield $\hat{\phi}$ is a scalar and transforms as

$$\delta\hat{\phi} = \Sigma^M \partial_M \hat{\phi}\,. \tag{2.1}$$

The fields $\hat{e}^A_M$ and $\hat{B}_{MN}$ transform as a vector and a two-index tensor respectively,

$$\begin{aligned} \delta\hat{e}^A_M &= \Sigma^P \partial_P \hat{e}^A_M + \partial_M \Sigma^P \hat{e}^A_P \\ \delta\hat{B}_{MN} &= \Sigma^P \partial_P \hat{B}_{MN} + \left(\partial_M \Sigma^P \hat{B}_{PN} - (-1)^{MN} \partial_N \Sigma^P \hat{B}_{PM}\right), \end{aligned} \tag{2.2}$$

and similarly for the RR superfields $\hat{C}, \hat{C}_{MN}, \hat{C}_{MNPQ}$. We denote the action of a (super) local Lorentz transformation on the vierbein as,

$$\delta\hat{e}^A_M = \Lambda^A{}_B \hat{e}^B_M\,. \tag{2.3}$$

There are additional gauge symmetries under which the the NS-NS two- form and the RR fields transform, these are superspace generalisations of the familiar gauge symmetries that act on the component supergravity fields. For example there is a gauge symmetry under which the $\hat{B}_{MN}$ transforms as,

$$\delta\hat{B}_{MN} = \partial_M \Sigma_N^{(b)} - (-1)^{MN}\partial_N \Sigma_M^{(b)}\,, \tag{2.4}$$

while the other fields are invariant. Similarly, there are gauge symmetries under which $\hat{C}_{MN}$ and $\hat{C}_{MNPQ}$ transform with gauge transformation parameters $\Sigma_M^{(c)}$ and $\Sigma_{MNP}^{(c)}$ respectively.

To zeroth order in θ we have the following identification of the superfields in terms of the component fields.

$$\begin{aligned} \hat{\phi} &= \phi \\ \hat{C} &= C \\ \hat{e}_m^a &= e_m^a \\ \hat{e}_m^\alpha &= \psi_m{}^\alpha \\ \hat{e}_\mu^\alpha &= \delta_\mu{}^\alpha \\ \hat{B}_{mn} &= B_{mn} \\ \hat{C}_{mn} &= C_{mn} \\ \hat{C}_{mnpq} &= C_{mnpq}\,, \end{aligned} \tag{2.5}$$

and all other fields are zero.

2.1 The dilaton superfield to $O(\theta^2)$

We are now ready to illustrate how the procedure of gauge completion works. We will first examine the dilaton superfield $\hat{\phi}$. Consider a super-diffeomorphism which to lowest order in θ has components,

$$\Sigma^m = 0\,, \qquad \Sigma^\alpha = \epsilon^\alpha\,. \tag{2.6}$$

From eq. (2.1) we see that, to $O(\theta^0)$, $\hat{\phi}$ transforms under this super-diffeomorphism as

$$\delta\hat{\phi} = \epsilon^\alpha \partial_\alpha \hat{\phi}\,. \tag{2.7}$$

Now since the lowest component of $\hat{\phi}$ is the dilaton, ϕ, we also know from the supersymmetry transformations of IIB supergravity fields (appendix A.1) that to this order,

$$\delta\hat{\phi} = \delta\phi = \bar{\epsilon}\lambda\,. \tag{2.8}$$

Equating these two expressions tells us that to $O(\theta^1)$, $\hat{\phi}$ is given by

$$\hat{\phi} = \phi + \bar{\theta}\lambda\,. \tag{2.9}$$

The components of super diffeomorphism we started with, eq. (2.6), are corrected at $O(\theta^1)$. We need to calculate these corrections as the first step in obtaining the $O(\theta^2)$ terms in $\hat{\phi}$. This can be done by relating the commutator of two supersymmetry transformations to translations.

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Given two supersymmetry transformations with parameters ϵ^1, ϵ^2 it is straightforward to see that the dilaton transforms under their commutator by a translation,

$$(\delta_1\delta_2 - \delta_2\delta_1)\phi = \xi^m \partial_m \phi \,, \tag{2.10}$$

where the translation parameter ξ^m is given by

$$\xi^m = \bar{\epsilon}_2 \Gamma^m \epsilon_1 \,. \tag{2.11}$$

On the other hand, from eq. (2.1) we see that the dilaton superfield under the commutator must transform as,

$$(\delta_1\delta_2 - \delta_2\delta_1)\hat{\phi} = \Sigma_2^P \partial_P \Sigma_1^M \partial_M \hat{\phi} - \Sigma_1^P \partial_P \Sigma_2^M \partial_M \hat{\phi} \,. \tag{2.12}$$

Requiring eq. (2.12) to agree with eq. (2.11) upto $O(\theta^0)$ allows us to obtain the $O(\theta^1)$ corrections to the super diffeomorphism, eq. (2.6).

We are interested in this paper in backgrounds where only bosonic supergravity fields acquire expectation values and not the fermionic fields ψ_μ and λ. With this in mind, from now on we will set terms depending on fermionic background fields to zero in the appropriate equations. To $O(\theta^1)$ one then finds that the components of the superdiffeomorphism are given by

$$\Sigma^m = \frac{1}{2}\bar{\theta}\Gamma^m \epsilon \,, \qquad \Sigma^\alpha = \epsilon^\alpha \,. \tag{2.13}$$

Actually, the general solution for Σ^M involves certain undetermined θ independent tensors. However, as explained elaborately in [6], by a redefinition of the superspace coordinates we can set them to zero resulting in eq. (2.13).

The $O(\theta^2)$ terms in the dilaton superfield, $\hat{\phi}$ can now be obtained by matching eq. (2.1) with

$$\delta\hat{\phi} = \delta\phi + \bar{\theta}\delta\lambda \,. \tag{2.14}$$

Using the expression for $\delta\lambda$ as given in the appendix A.1, we find

$$\hat{\phi} = \phi + \bar{\theta}\lambda - \frac{1}{48}\bar{\theta}\Gamma^{mnp}\sigma^3\theta H_{mnp} - \frac{1}{4}e^\phi\bar{\theta}\Gamma^m(i\sigma^2)\theta F_m - \frac{1}{48}e^\phi\bar{\theta}\Gamma^{mnp}\sigma^1\theta F_{mnp} \,. \tag{2.15}$$

2.2 $\hat{B}_{MN}$ to $O(\theta^2)$

We now turn to the NS-NS two-form superfield $\hat{B}_{MN}$. The only new twist here is that we will need to include a suitable gauge transformation, eq. (2.4), with the coordinate transformation, eq. (2.13), to determine the θ expansion in this case.

To understand this let us first calculate the commutator of two supersymmetry transformations, with parameters, ϵ^1, ϵ^2 on the component supergravity field B_{mn}. Using the susy transformation rules given in the appendix A.1,

$$\delta_1\delta_2 B_{mn} = \bar{\epsilon}_2\sigma^3\left(\Gamma_m\delta_1\psi_n - \Gamma_n\delta_1\psi_m\right), \tag{2.16}$$

we find that

$$(\delta_1\delta_2 - \delta_2\delta_1)B_{mn} = -\big(\partial_m(\bar\epsilon_2\sigma^3\Gamma_n\epsilon_1) - \partial_n(\bar\epsilon_2\sigma^3\Gamma_m\epsilon_1)\big) + \bar\epsilon_2\Gamma^p\epsilon_1 H_{mnp}$$
$$= \xi^p\partial_p B_{mn} + \partial_m\xi^p B_{pn} - \partial_n\xi^p B_{pm} + \partial_m\xi_n^{12(b)} - \partial_n\xi_m^{12(b)}\,. \quad (2.17)$$

The second line on the r.h.s. is the transformation of B_{mn} under a combined translation by ξ^m and a gauge transformation with parameter $\xi_n^{12(b)}$. One finds that this equation can be met if ξ^n is given by eq. (2.11), and the gauge transformation parameter is,

$$\xi_m^{12(b)} = \xi^n B_{mn} - \bar\epsilon_2\sigma^3\Gamma_m\epsilon_1\,. \quad (2.18)$$

In terms of superfields this tells us that the super-diffeomorphism, eq. (2.13), should be accompanied by a gauge transformation. We denote the gauge transformation parameter in superspace by $\Sigma^{(b)}$ below. The combined transformation can then be written as,

$$\delta\hat B_{MN} = \Sigma^P\partial_P\hat B_{MN} + \partial_M\Sigma^P B_{PN} - (-1)^{MN}\partial_N\Sigma^P B_{PM} + \partial_M\Sigma_N^{(b)} - (-1)^{MN}\partial_N\Sigma_M^{(b)}\,. \quad (2.19)$$

The commutator of two transformations in superspace can now be calculated. We get that

$$(\delta_1\delta_2 - \delta_2\delta_1)\hat B_{MN} = \partial_M\Sigma_N^{12(b)} - (-1)^{MN}\partial_N\Sigma_M^{12(b)} + \cdots\,. \quad (2.20)$$

The ellipses on the rhs denote extra terms which arise due to a coordinate transformation with parameters, eq. (2.13). $\Sigma^{12(b)}$ above denotes a gauge transformation, it's components turn out to be,

$$\Sigma_M^{12(b)} = \Big(\Sigma_2^P\partial_P\Sigma_M^{1(b)} + \partial_M\Sigma_2^P\Sigma_P^{1(b)}\Big) - (1\leftrightarrow 2)\,. \quad (2.21)$$

To leading order in θ, $B_{m\mu}$ and $B_{\mu\nu}$ both vanish and the only non-zero component of $\hat B_{MN}$ is B_{mn}. Comparing eq. (2.21) and eq. (2.18) and using eq. (2.13) for the components of Σ^P we then find that upto $O(\theta)$,

$$\Sigma_m^{(b)} = \frac{1}{2}\bar\theta\left(\Gamma^n B_{mn} - \sigma^3\Gamma_m\right)\epsilon\,. \quad (2.22)$$

And $\Sigma_\mu^{(b)} = 0$.

We are now ready to evaluate $\hat B_{MN}$ to higher orders in θ. From the susy transformation, appendix A.1, for the supergravity field B_{mn} it follows that upto $O(\theta^1)$

$$\hat B_{mn} = B_{mn} + \bar\theta\sigma^3\Gamma_m\psi_n - \bar\theta\sigma^3\Gamma_n\psi_m\,. \quad (2.23)$$

To evaluate $\hat B_{m\mu}$, note that

$$\delta\hat B_{m\mu} = \epsilon^\alpha\partial_\alpha\hat B_{m\mu} + \frac{1}{2}\left(\bar\epsilon\sigma^3\Gamma_m\right)_\mu \quad (2.24)$$

Since $B_{m\mu}$ vanishes at zeroth order in θ, the above variation should be zero, which gives

$$\hat B_{m\mu} = -\frac{1}{2}\left(\bar\theta\sigma^3\Gamma_m\right)_\mu\,. \quad (2.25)$$

Similarly one can show that $\hat B_{\mu\nu}$ must vanish upto $O(\theta^1)$.

To find the second order results for $\hat{B}_{mn}$, we consider the variation of $\hat{B}_{mn}$, eq. (2.19), upto to first order in θ. Using the results for the superdiffeomorphism, eq. (2.13), and gauge transformation, eq. (2.22), this is given by

$$\delta\hat{B}_{mn} = \epsilon^\alpha\partial_\alpha\hat{B}_{mn} + \frac{1}{2}\bar{\theta}\Gamma^p\epsilon H_{mnp} + \bar{\theta}\sigma^3\left(\Gamma_m\partial_n\epsilon - \Gamma_n\partial_m\epsilon\right) - \frac{1}{2}\bar{\theta}\sigma^3\Gamma_a\epsilon\left(\partial_m e_n{}^a - \partial_n e_m{}^a\right). \tag{2.26}$$

On the other hand this has to be equated with the variation of eq. (2.23) leading to

$$\begin{aligned}\delta\hat{B}_{mn} &= \delta B_{mn} + \bar{\theta}\sigma^3\left(\Gamma_m\delta\psi_n - \Gamma_n\delta\psi_m\right)\\ &= \bar{\epsilon}\sigma^3\left(\Gamma_m\psi_n - \Gamma_n\psi_m\right) + \bar{\theta}\sigma^3\left(\Gamma_m\partial_n\epsilon - \Gamma_n\partial_m\epsilon\right) - \frac{1}{2}\bar{\theta}\sigma^3\Gamma_a\epsilon\left(\partial_m e_n{}^a - \partial_n e_m{}^a\right) +\\ &\quad + \frac{1}{4}\bar{\theta}\sigma^3\left(\omega_m{}^{ab}\Gamma_{nab} - \omega_n{}^{ab}\Gamma_{mab}\right)\epsilon - \frac{1}{4}e^\phi\bar{\theta}\sigma^1\Gamma_{mnp}\epsilon F^p + \frac{1}{2}\bar{\theta}\Gamma^p\epsilon H_{mnp} -\\ &\quad - \frac{1}{8}\bar{\theta}\left(\Gamma_m{}^{pq}H_{npq} - \Gamma_m{}^{pq}H_{mpq}\right)\epsilon - \frac{1}{24}e^\phi\bar{\theta}(i\sigma^2)\left(\Gamma_{mn}{}^{pqr}F'_{pqr} + 6\Gamma^p F'_{mnp}\right)\epsilon -\\ &\quad - \frac{1}{8\cdot 5!}e^\phi\bar{\theta}\sigma^1\left(\Gamma_{mn}{}^{pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{mnpqr}\right)\epsilon\,,\end{aligned} \tag{2.27}$$

where on the rhs we have used the susy transformations for B_{mn} and the gravitino from appendix A.1. Eq. (2.26), (2.27) finally give us the expansion to second order in θ,

$$\begin{aligned}\hat{B}_{mn} &= B_{mn} + \bar{\theta}\sigma^3\left(\Gamma_m\psi_n - \Gamma_n\psi_m\right) + \frac{1}{8}\bar{\theta}\sigma^3\left(\omega_m{}^{ab}\Gamma_{nab} - \omega_n{}^{ab}\Gamma_{mab}\right)\theta -\\ &\quad - \frac{1}{16}\bar{\theta}\left(\Gamma_m{}^{pq}H_{npq} - \Gamma_m{}^{pq}H_{mpq}\right)\theta -\\ &\quad - \frac{1}{8}e^\phi\bar{\theta}\sigma^1\Gamma_{mnp}\theta F^p - \frac{1}{48}e^\phi\bar{\theta}(i\sigma^2)\left(\Gamma_{mn}{}^{pqr}F'_{pqr} + 6\Gamma^p F'_{mnp}\right)\theta -\\ &\quad - \frac{1}{16\cdot 5!}e^\phi\bar{\theta}\sigma^1\left(\Gamma_{mn}{}^{pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{mnpqr}\right)\theta\,.\end{aligned} \tag{2.28}$$

As was mentioned in the discussion of the previous subsection we are interested in backgrounds for which the fermions ψ_m and λ are zero. Also, when we construct the world volume action it will be convenient to work in static gauge and fix the κ-symmetry by choosing the space time spinors θ_1, θ_2 to be

$$\theta_1 = \Theta\,, \qquad \theta_2 = 0\,. \tag{2.29}$$

With this choice the expression for $\hat{B}_{mn}$ becomes

$$\hat{B}_{mn} = B_{mn} + \frac{1}{8}\bar{\Theta}\left(\omega_m{}^{ab}\Gamma_{nab} - \omega_n{}^{ab}\Gamma_{mab}\right)\Theta - \frac{1}{16}\bar{\Theta}\left(\Gamma_m{}^{pq}H_{npq} - \Gamma_m{}^{pq}H_{mpq}\right)\Theta\,. \tag{2.30}$$

It will be enough for our purposes of determining the fermion bilinear terms below to determine the other components $B_{m\mu}, B_{\mu\nu}$, to $O(\theta^1)$ which was already done above.

2.3 Final results

One can follow through similar steps to obtain the expansions for other superfield. We have summarised the results below, detail calculations are performed in the appendix.

As was mentioned above, we have set the fermionic backgrounds to zero. Also we work with the choice of spinors in eq. (2.29). The components of the superfields to required order in the θ expansion are then given by:

$$\begin{aligned}
\hat{\phi} &= \phi - \frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp} \\
\hat{C} &= C - \frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta F'_{mnp} \\
\hat{e}^a_\mu &= -\frac{1}{2}\left(\bar{\theta}\Gamma^a\right)_\mu \\
\hat{e}^a_m &= e_m{}^a - \frac{1}{8}\omega_m{}^{cd}\bar{\Theta}\Gamma^a{}_{cd}\Theta - \frac{1}{16}H_{mpq}\bar{\Theta}\Gamma^{apq}\Theta \\
\hat{B}_{m\mu} &= -\frac{1}{2}\left(\bar{\theta}\sigma^3\Gamma_m\right)_\mu \\
\hat{B}_{mn} &= B_{mn} - \frac{1}{8}\bar{\Theta}\left(\Gamma_m{}^{ab}\omega_{nab} - \Gamma_n{}^{ab}\omega_{mab}\right)\Theta - \frac{1}{16}\bar{\Theta}\left(\Gamma_m{}^{pq}H_{npq} - \Gamma_n{}^{pq}H_{mpq}\right)\Theta \\
\hat{C}_{m\mu} &= \frac{1}{2}e^{-\phi}\left(\bar{\theta}\sigma^1\Gamma_m\right)_\mu - \frac{1}{2}C\left(\bar{\theta}\sigma^3\Gamma_m\right)_\mu \\
\hat{C}_{mn} &= C_{mn} - \frac{1}{8}C\bar{\Theta}\left(\Gamma_m{}^{ab}\omega_{nab} - \Gamma_n{}^{ab}\omega_{mab}\right)\Theta + \frac{1}{8}\bar{\Theta}\Gamma_{mnp}\Theta F^p - \\
&\quad - \frac{1}{16}C\bar{\Theta}\left(\Gamma_m{}^{pq}H_{npq} - \Gamma_n{}^{pq}H_{mpq}\right)\Theta - \frac{1}{16}\bar{\Theta}\left(\Gamma_m{}^{pq}F'_{npq} - \Gamma_n{}^{pq}F'_{mpq}\right)\Theta - \\
&\quad - \frac{1}{16\cdot 5!}\bar{\Theta}\left(\Gamma_{mn}{}^{pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{mnpqr}\right)\Theta \\
\hat{C}_{\mu mnp} &= -\frac{1}{2}e^{-\phi}\left(\bar{\theta}(i\sigma^2)\Gamma_{mnp}\right)_\mu + \frac{3}{2}\left(\bar{\theta}\sigma^3 C_{[mn}\Gamma_{p]}\right)_\mu \\
\hat{C}_{mnpq} &= C_{mnpq} - \frac{3}{2}\bar{\Theta}C_{[mn}\Gamma_p{}^{ab}\omega_{q]ab}\Theta - \frac{3}{4}\bar{\Theta}C_{[mn}\Gamma_p{}^{st}H_{q]st}\Theta + \\
&\quad + \bar{\Theta}\left(\frac{1}{48}\Gamma_{mnpq}{}^{stu}F'_{stu} + \frac{1}{2}\Gamma_{[mnp}F_{q]} + \frac{3}{4}\Gamma_{[mn}{}^{s}F'_{pq]s} - \right. \\
&\qquad \left. - \frac{1}{96}\Gamma_{[mnp}{}^{stuv}F'_{q]stuv} - \frac{1}{8}\Gamma_{[m}{}^{st}F'_{npq]st}\right)\Theta\,.
\end{aligned} \tag{2.31}$$

Here, $H_3 = dB$. And F'_{mnp}, F'_{mnpqrs}, refer to the components of the three form, $dC_2 - C_0 H_3$, and the five form, $dC_4 - C_2 \wedge H_3$, respectively. Eq. (2.31) is one of the main results of our paper.

3. World volume action

3.1 The action

The action for the $D3$ brane is given by [12 – 16]

$$S = -\mu_3 \int d^4\zeta e^{-\hat{\phi}}\sqrt{-\det\left(\hat{g}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} + F_{\tilde{i}\tilde{j}} - \hat{B}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}}\right)} + \mu_3 \int e^{F-\hat{B}} \wedge \hat{\mathbf{C}}\,. \tag{3.1}$$

It is obtained by pulling back the superfields from spacetime to the $D3$ brane world volume. For on-shell background fields the action is κ-symmetric. In eq. (3.1) $\zeta^{\tilde{i}}, \tilde{i} = 0, \dots 3$ are the world volume coordinate. We also denote $\hat{\mathbf{C}} = \oplus_n \hat{C}_n$.

It will be useful in the discussion below to distinguish between the pullback of the superfield and pullback of the component bosonic supergravity fields. The pullback of a superfield is by definition obtained by pulling back the superspace tensor onto the world-volume. For example, the pullback of $\hat{B}_{MN}$ is,

$$\hat{B}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} = \partial_{\tilde{i}} Z^M \partial_{\tilde{j}} Z^N \hat{B}_{MN} \,, \tag{3.2}$$

where $Z^M = (x^m, \theta^\mu)$ are the spacetime superspace coordinates. This is what appears in eq. (3.1). In contrast we define the pullback of the component supergravity field from the ordinary (Bosonic) target space to the worldvolume. So,

$$B_{\tilde{i}\tilde{j}} = \partial_{\tilde{i}} x^m \partial_{\tilde{j}} x^n B_{mn} \,. \tag{3.3}$$

To distinguish between the two we will use boldface indices in the case of the superfield, as in eq. (3.1), (3.2) above.

It will also be useful to distinguish between the lowest order term and the higher order contributions in the θ expansion for any superfield. the latter will be denoted by an additional prime. For example, we can write for the dilaton superfield,

$$\hat{\phi} = \phi + \phi' \tag{3.4}$$

where from eq. (2.31), $\phi' = -\frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp}$.

Using the expressions for the super vierbeins from eq. (2.31), it then follows that the metric $\hat{g}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} = \hat{e}^a_{\tilde{\mathbf{i}}}\hat{e}^b_{\tilde{\mathbf{j}}}\eta_{ab}$ to second order in Θ is,

$$\hat{g}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} = g_{\tilde{i}\tilde{j}} + \left(\partial_{\tilde{i}} x^m \partial_{\tilde{j}} x^n + \partial_{\tilde{j}} x^m \partial_{\tilde{i}} x^n\right) e^b_n e'^a_m \eta_{ab} + \frac{1}{2}\bar{\Theta}\Gamma^a \left(D_{\tilde{i}}\Theta\partial_{\tilde{j}} x^m + D_{\tilde{j}}\Theta\partial_{\tilde{i}} x^m\right) e^b_m \eta_{ab} \,. \tag{3.5}$$

A similar straightforward analysis shows that the pull back of the NS and RR superfields become

$$\begin{aligned}
\hat{B}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} &= B_{\tilde{i}\tilde{j}} + \partial_{\tilde{i}} x^m \partial_{\tilde{j}} x^n B'_{mn} + \frac{1}{2}\left(\bar{\Theta}\Gamma_{\tilde{i}} D_{\tilde{j}}\Theta - \bar{\Theta}\Gamma_{\tilde{j}} D_{\tilde{i}}\Theta\right)\\
\hat{C}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}} &= C_{\tilde{i}\tilde{j}} + \partial_{\tilde{i}} x^m \partial_{\tilde{j}} x^n C'_{mn} + \frac{1}{2} C\left(\bar{\Theta}\Gamma_{\tilde{i}}\partial_{\tilde{j}}\Theta - \bar{\Theta}\Gamma_{\tilde{j}}\partial_{\tilde{i}}\Theta\right)\\
\hat{C}_{\tilde{\mathbf{i}}\tilde{\mathbf{j}}\tilde{\mathbf{k}}\tilde{\mathbf{l}}} &= C_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}} + \partial_{\tilde{i}} x^m \partial_{\tilde{j}} x^n \partial_{\tilde{k}} x^p \partial_{\tilde{l}} x^q C'_{mnpq} + 4!\partial_{[\tilde{i}}\Theta^\mu \partial_{\tilde{j}} x^n \partial_{\tilde{k}} x^p \partial_{\tilde{l}]} x^q \hat{C}_{\mu npq} \,.
\end{aligned} \tag{3.6}$$

Using these expressions we can compute the world volume action. The DBI action becomes

$$\begin{aligned}
S_{DBI} = -\mu_3 \int d^4\zeta e^{-\phi}\sqrt{\det g}\Bigg\{&\left(1 + \frac{1}{4}(F-B)^2\right)\left(1 + \frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp}\right) + \\
&+ \frac{1}{2}\left(\delta_{\tilde{i}}^{\tilde{k}} + (F-B)_{\tilde{i}}^{\tilde{k}}\right)\left(\bar{\Theta}\Gamma_{\tilde{k}} D^{\tilde{i}}\Theta - \frac{1}{8}\bar{\Theta}\Gamma_{\tilde{k}pq}\Theta H^{\tilde{i}pq}\right) + \cdots\Bigg\} \,.
\end{aligned} \tag{3.7}$$

Here we have followed a slightly condensed notation. In our notation above, $\tilde{i}, \tilde{j}, \tilde{k}$ denote world volume indices, whereas m, n, p denote spacetime (bosonic) indices. Now, $\Gamma_{\tilde{k}} \equiv \Gamma_m \partial_{\tilde{k}} x^m$, $\partial^{\tilde{i}}\Theta \equiv g^{\tilde{i}\tilde{j}}\partial_{\tilde{j}}\Theta$, etc. The ellipses on the right hand side above indicate additional terms that can be obtained by expanding the square root in eq. (3.1) to higher order. In addition of course extra terms would arise if we carried out the θ expansion of the superfields to higher order as well.

Similarly the Wess-Zumino action is

$$
\begin{aligned}
S_{WZ} = \mu_3 \int e^{(F-B)} \wedge \mathbf{C} - \frac{1}{96}\mu_3 \int (F-B)\wedge(F-B)\bar{\Theta}\Gamma^{mnp}\Theta F'_{mnp} + \\
+\frac{1}{32}\mu_3 \int d^4\zeta \sqrt{\det g}\,\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}(F-B)_{\tilde{i}\tilde{j}} \times \\
\times\bar{\Theta}\Bigg\{\Gamma_{\tilde{k}\tilde{l}p}F^p - \Gamma_{\tilde{k}}{}^{pq}F'_{\tilde{l}pq} - \frac{1}{2\cdot 5!}\Big(\Gamma_{\tilde{k}\tilde{l}}{}^{pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{\tilde{k}\tilde{l}pqr}\Big)\Bigg\}\Theta + \\
+\frac{1}{16}\mu_3 \int d^4\zeta \sqrt{\det g}\,\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Big(\frac{1}{72}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}{}^{pqr}\Theta F'_{pqr} + \frac{1}{3}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}}\Theta F_{\tilde{l}} + \\
+\frac{1}{2}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}}{}^{p}\Theta F'_{\tilde{k}\tilde{l}p} - \frac{1}{3!}\bar{\Theta}\Gamma_{\tilde{i}}{}^{pq}\Theta F'_{\tilde{j}\tilde{k}\tilde{l}pq}\Big). \qquad (3.8)
\end{aligned}
$$

Equations, (3.7) and (3.8), are important for the the following discussion. We will see in the next section that the action above agrees with other established results in the literature.

3.2 Some comments

Two comments are now in order.

One of the main motivations for this project is to understanding non-perturbative corrections to the superpotential which can arise in flux compactifications. In this context we are interested in IIB string theory compactified down to $R^{3,1}$ (actually for the non-perturbative corrections we are interested in the euclidean situation R^4 as discussed in the next section). One class of non-perturbative effects, which is our main focus here, arise due to to euclidean D3 branes that wrap a holomorphic 4-cycle, i.e. a divisor, in the internal 6-dimensional space.

Under a duality map to M-theory this lifts to a euclidean 5-brane instanton wrapping a divisor of the Calabi-Yau four-fold. The resulting superpotential was discussed in the seminal paper of Witten [3]. An U(1) symmetry played an important role in this analysis. This symmetry is a subgroup of the structure group of the normal bundle and corresponds to rotations in the plane of the two compact directions orthogonal to the divisor. An index was formulated by counting the fermionic zero modes after grading them by their charge under this symmetry. This index turned out to be proportional to the arithmetic genus of the divisor and it was argued that a correction could only arise if the arithmetic genus was unity.

In the IIB description we are using here the U(1) the divisor is 2 complex dimensional and the compactified space is 6-dimensional. This means, roughly speaking, that two compact directions are normal to the divisor and the U(1) symmetry is rotations in the plane formed by these two directions. We will now see that the presence of three-form flux can lead to this U(1) symmetry being broken in the D3-brane world volume theory. As a result, zero modes with the *same* U(1) charge can pair up and get heavy. In this way, a correction to the superpotential can arise even though the index condition mentioned above is not met.

The essential point is simply that if the three form flux has two legs along the 4-cycle and one perpendicular to it then it will break the U(1) symmetry mentioned above. Since the fluxes enter in various bilinear fermion couplings in eqs. (3.7) and (3.8), the mass terms for the fermions will in general violate this symmetry. To illustrate this concretely let us consider the situation where $F-B$ in the world-volume theory vanishes. Then the fermion three-form flux dependent mass terms for a D3 brane wrapping a 4-cycle take the form,

$$S_{\text{mass}} = -\mu_3 \int d^4\xi \sqrt{\text{det}g}\bar{\Theta} \left\{ e^{-\phi}\frac{1}{48}\Gamma^{mnp}H_{mnp} - \frac{1}{16}e^{-\phi}\Gamma_{\tilde{i}pq}H^{\tilde{i}pq} - \frac{1}{32}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Gamma^{p}_{\tilde{i}\tilde{j}}F'_{\tilde{k}\tilde{l}p} \right\} \Theta \tag{3.9}$$

(we have used the fact that the flux preserves Poincare invariance in $R^{3,1}$ to set some terms to zero). We remind the reader that in our notation, indices, $\tilde{i}, \tilde{j}, \tilde{k}, \tilde{l}$ are along the worldvolume, and m, n, p take $0, \ldots, 9$ values in spacetime. Now, in general, it is easy to see that if H, F' have two legs along the brane and one along the normal then each of the terms appearing above breaks the U(1) symmetry. Also, the sum of these terms does not vanish for on-shell backgrounds, even those which meet the conditions of supersymmetry. Thus, as was mentioned above the mass terms will in general break the U(1) symmetry allowing in particular two fermions with same sign charges to pair up and get heavy.

Second, let us now consider the special case of a $D3$-brane which is along $R^{3,1}$ and transverse to the internal directions. We also take the background fields to preserve the Poincare symmetry of $R^{3,1}$. In addition, we take the space time metric to be of the form $g_{10} = e^{2A(y^m)}\eta_4 \otimes g_6^{tr}$. The DBI term is then given by,

$$\begin{aligned} S_{DBI} = -\mu_3 \int d^4\zeta e^{-\phi}\sqrt{\text{det}g} \Bigg\{ & \left(1 + \frac{1}{4}(F-B)^2\right)\left(1 + \frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp}\right) + \\ & + \frac{1}{2}\left(\delta_{\tilde{i}}^{\ \tilde{k}} + (F-B)_{\tilde{i}}^{\ \tilde{k}}\right)\bar{\Theta}\Gamma_{\tilde{k}}\partial^{\tilde{i}}\Theta + \cdots \Bigg\}. \end{aligned} \tag{3.10}$$

The spin connection dependent term vanishes in the above equation for the a general warped metric. The Wess-Zumino term is given by

$$\begin{aligned} S_{WZ} = \mu_3 \int C_4 & - \frac{1}{96}\mu_3 \int (F-B)\wedge(F-B)\bar{\Theta}\Gamma^{mnp}\Theta F'_{mnp} + \\ & + \frac{1}{32}\mu_3 \int d^4\zeta \sqrt{\text{det}g}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}(F-B)_{\tilde{i}\tilde{j}} \times \\ & \times \bar{\Theta}\left\{ \Gamma_{\tilde{k}\tilde{l}p}F^p - \frac{1}{2\cdot 5!}\left(\Gamma_{\tilde{k}\tilde{l}}^{\ \ pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{klpqr}\right)\right\}\Theta + \\ & + \frac{1}{48\cdot 4!}\mu_3 \int d^4\zeta \sqrt{\text{det}g}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}^{\ \ \ \ pqr}\Theta F'_{pqr}\,. \end{aligned} \tag{3.11}$$

The full action is the sum of these two terms. This result is of interest from the point of view of calculating the soft terms that can arise after turning on fluxes [20−27]. It agrees (upto some minor discrepancy in the numerical factors) with ref. [7].

Ignoring terms dependent on $(F-B)$, the $O(\Theta^2)$ part of the action becomes

$$S(\Theta^2) = \frac{\mu_3}{48}\int d^4\zeta e^{-\phi}\sqrt{\text{det}g}\bar{\Theta}\Gamma^{mnp}\Theta\text{Re}(*G - iG)_{mnp}\,, \tag{3.12}$$

where $G \equiv F' - ie^{-\phi}H$. We see that for imaginary self dual flux, the above term vanishes. This is to be expected from the analysis of [28].

4. T-duality and comparison with other results

As a simple check of our results, we can take the type-IIA action for D0 brane and perform three T-dualities to obtain the action for D3 brane. The D0 brane action to order Θ^2, in the Einstein frame, is given by [8]

$$\begin{aligned} S = & -\mu_0 \int d\tau e^{-\frac{3}{4}\phi} \left(1 - \frac{3}{4}\Phi|_{\Theta^2} + \cdots\right) \sqrt{-\left(g_{mn} + 2e_{ma}E^a_n|_{\Theta^2} + \cdots\right)\dot{x}^m\dot{x}^n} + \\ & +\mu_0 \int d\tau \left(C_m + B_m|_{\Theta^2} + \cdots\right)\dot{x}^m \end{aligned} \tag{4.1}$$

where the dots indicate terms of higher order in Θ. The order Θ^2 part of the IIA superfields are given by

$$\begin{aligned} \Phi|_{\Theta^2} &= \frac{i}{48}e^{-\frac{1}{2}\phi}\bar{\Theta}\Gamma^{mnp}\Theta G_{mnp} \\ B_m|_{\Theta^2} &= -\frac{i}{16}\bar{\Theta}\gamma_m{}^{np}\Theta F_{np} - \frac{i}{48}e^{-\frac{1}{2}\phi}\bar{\Theta}\gamma^{npq}F'_{mnpq} \\ E^a_m|_{\Theta^2} &= \frac{i}{8}\bar{\Theta}\gamma^{abc}\Theta\omega_{mbc} + \frac{i}{64}e^{-\frac{1}{2}\phi}\left(\bar{\Theta}\gamma_m{}^{np}\Theta H^r{}_{np} + 3\bar{\Theta}\gamma^{anp}\Theta H_{mnp} - \frac{1}{3}e^a_m\bar{\Theta}\gamma^{npq}\Theta H_{npq}\right). \end{aligned} \tag{4.2}$$

Using the above formulae, we write the action in terms component fields. Also, before performing T-duality, we make the following field redefinitions [7] to change the action in to sting frame.

$$g_{mn(E)} = e^{-\frac{1}{2}\phi}g_{mn(S)}\,, \qquad \Gamma^m_{(E)} = e^{\frac{1}{4}\phi}\Gamma^m_{(S)}\,, \qquad \Theta_{(E)} = e^{-\frac{1}{8}\phi}\Theta_{(S)}\,. \tag{4.3}$$

With this, the DBI action becomes

$$S_{DBI} = -\mu_0 \int d\tau e^{-\phi}\sqrt{-g_{00}}\left(1 + \frac{i}{8}\bar{\Theta}\left\{\gamma^{0ab}\omega_{0ab} + \frac{1}{2}\gamma^{0np}H_{0np} - \frac{1}{6}\gamma^{mnp}H_{mnp}\right\}\Theta + \cdots\right) \tag{4.4}$$

and the Wess-Zumino part

$$S_{WZ} = \mu_0 \int d\tau \left\{C_0 - \frac{i}{16}\left(\bar{\Theta}\gamma_0{}^{mn}\Theta F_{mn} + \frac{1}{3}\bar{\Theta}\gamma^{mnp}\Theta F'_{0mnp}\right) + \cdots\right\}. \tag{4.5}$$

Now we perform three T-dualities along $\{x, y, z\}$. Let us denote these directions by $\acute{m}, \acute{n}, \ldots$ and the remaining directions by $\check{p}, \check{q}, \ldots$. For simplicity, we consider the following special case. We assume $g_{\acute{m}\check{p}} = B_{\acute{n}\check{q}} = B_{\acute{n}\acute{m}} = 0$ and we take the metric along the directions x, y, z to be diagonal. Also we set the spin connection to zero. Using the T-duality rules as given in the appendix A.5, it is then straightforward to see that the quadratic part of the action (4.4) is identical to our result

$$S_{DBI}(\Theta^2) = -\mu_3 \int d^4\zeta e^{-\phi}\sqrt{\det g}\left(\frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp} - \frac{1}{16}\bar{\Theta}\Gamma_{\tilde{i}pq}\Theta H^{\tilde{i}pq}\right). \tag{4.6}$$

We can now turn to the quadratic part of the Wess-Zumino action. After performing the duality, we find

$$
\begin{aligned}
&-\frac{i}{16}\left(\bar{\Theta}\gamma_0{}^{mn}\Theta F_{mn}+\frac{1}{3}\bar{\Theta}\gamma^{mnp}\Theta F'_{0mnp}\right)=\\
&=\bar{\Theta}\Big\{\frac{i}{16}\gamma_0{}^{\check{p}\check{q}}F_{xyz\check{p}\check{q}}+\frac{i}{8}(\gamma_{0x}{}^{\check{p}}F_{yz\check{p}}+\gamma_{0y}{}^{\check{p}}F_{zx\check{p}}+\gamma_{0z}{}^{\check{p}}F_{xy\check{p}})-\frac{i}{48}\gamma^{\check{p}\check{q}\check{r}}F'_{0xyz\check{p}\check{q}\check{r}}+\\
&\quad+\frac{i}{8}(\gamma_{xyz}F_0-\gamma_{0xy}F_z-\gamma_{0yz}F_x-\gamma_{0zx}F_y)-\\
&\quad-\frac{i}{16}(\gamma_x{}^{\check{p}\check{q}}F'_{0yz\check{p}\check{q}}+\gamma_y{}^{\check{p}\check{q}}F'_{0zx\check{p}\check{q}}+\gamma_z{}^{\check{p}\check{q}}F'_{0xy\check{p}\check{q}})-\\
&\quad-\frac{i}{8}(\gamma_{xy}{}^{\check{p}}F'_{z0\check{p}}+\gamma_{yz}{}^{\check{p}}F'_{x0\check{p}}+\gamma_{zx}{}^{\check{p}}F'_{y0\check{p}})\Big\}\Theta
\end{aligned}
\tag{4.7}
$$

which coincides with the quadratic action

$$
\begin{aligned}
S_{WZ}=\mu_3\int d^4\zeta\sqrt{\det g}\,\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\bigg(&\frac{1}{4!\cdot 48}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}{}^{pqr}\Theta F'_{pqr}+\frac{1}{48}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}}\Theta F_{\tilde{l}}+\\
&+\frac{1}{32}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}}{}^{p}\Theta F'_{\tilde{k}\tilde{l}p}-\frac{1}{16\cdot 3!}\bar{\Theta}\Gamma_{\tilde{i}}{}^{pq}\Theta F'_{\tilde{j}\tilde{k}\tilde{l}pq}\bigg).
\end{aligned}
\tag{4.8}
$$

We have chosen the gauge, eq. (2.29), in constructing the D3 brane action. Agreement with the D0 brane action shows that this agrees with the gauge choice, $\Gamma^{11}\Theta=-\Theta$ in the IIA case. This point was already noted in [7].

As was mentioned in the introduction the action for branes upto quadratic order in fermions in the presence of an arbitary on-shell background has already been derived by Marolf, Martucci and Silva [10, 11]. These authors used the method of "normal coordinate expansion" together with T-duality which is different from the method of gauge completion used here. As we discuss below our results completely agree. This constitutes a significant check of our results and methods.

The quadratic fermion terms in the action for a D_p brane are given in eq.(30) of [11]. We are interested in the case $p=3$ here. $\tilde{\Gamma}_{D_p}$ is defined in eq.(28) and L_p in eq.(29) of [11], with $\Gamma^{\phi}=-\sigma_3$ in our notation. Also, D_m and Δ are defined in eq.(84), (86) of [11]. y in eq.(30) of [11] stands for the 32 component spinor that we call θ, with y_1, y_2 corresponding to θ_1, θ_2 respectively . Let us for simplicity now set the world volume magnetic field to zero. In the gauge $y_2=0$, it is then easy to see that eq.(30) of [11] agrees completely with the fermion bilinear terms obtained above, eq. (3.7), eq. (3.8), after identifying y_1 with Θ and the RR field strengths with each other upto a sign.

Finally, the world volume action of $M5$ brane in presence of background flux has been constructed by Kallosh and Sorokin [9]. After a duality map this can be related to the D3 brane action computed here. We have compared with the fermion bilinear terms presented in eq.(22) of [9] and find substantial agrement.[2]

---

[2]We are greatful to R. Kallosh and D. Sorokin for help in this regards.

5. An example

5.1 Euclidean continuation

The discussion above was for a D3 brane in Minkowski space with signature $(9,1)$. Our main interest here is in instanton corrections to the superpotential and for this purpose we are really interested in euclidean space with signature $(10,0)$. We will not consider time dependent backgrounds here and continuing the bosonic fields which appear in the world volume theory eqs. (3.7) and (3.8), to euclidean space is straightforward. The world volume theory also contains a 16 component Majorana Weyl fermion, Θ. This is continued to a 16 component complex Weyl fermion in euclidean space.[3] Fermion bilinear terms of the form:

$$S=\int d^4\xi\Theta^T\Gamma^0 M\Theta\,, \tag{5.1}$$

are continued to euclidean space by replacing Θ above by the Weyl fermion. The path integral of the world-volume theory is then carried out over Θ alone. In particular $\Theta^\dagger$ does not appear in the path integral as an independent degree of freedom. In this way no further doubling of the fermionic degrees is introduced for the purposes of evaluating the path integral [29].

5.2 The example

Now let us consider a specific example that will illustrate the role that fluxes can play in changing the count of zero modes. We consider a T^6/Z_2 compactification with flux [30, 31]. The six coordinates of torus are taken to be, $x^i, y^i, i=1,\ldots,3$, with $0\le x^i, y^i\le 1$. The Z_2 orientifold symmetry involves a reflection in all six directions, $(x^i,y^i)\to-(x^i,y^i), i=1,\ldots,3$. Holomorphic coordinates are, $Z^i=x^i+\tau_{ij}y^j$, where τ_{ij} determine the complex structure of the torus. The tree-level superpotential takes the form, [32, 28],

$$W_{\text{tree}}=\int(F-\Phi H)\wedge\Omega_3 \tag{5.2}$$

where $\Phi=C+ie^{-\phi}$ is the axion-dilaton, and Ω_3 is the holomorphic three-form which in this case takes the form, $\Omega_3=dZ^1\wedge dZ^2\wedge dZ^3$.

We focus on one specific choice of flux: F and H:

$$\begin{aligned}F&=dx^1\wedge dx^2\wedge dx^3+dy^1\wedge dy^2\wedge dy^3\\ H&=dx^1\wedge dx^2\wedge dx^3-2dy^1\wedge dy^2\wedge dy^3-dx^2\wedge dx^3\wedge dy^1-dx^3\wedge dx^1\wedge dy^2-\\ &\quad-dx^1\wedge dx^2\wedge dy^3+dy^2\wedge dy^3\wedge dx^1+dy^3\wedge dy^1\wedge dx^2+dy^1\wedge dy^2\wedge dx^3\,.\end{aligned} \tag{5.3}$$

This example was analysed in [30] and it was shown that as a result of the superpotential, eq. (5.2), all the complex structure moduli of the torus as well as the axion-dilaton are stabilized with a value

$$C+ie^{-\phi}=e^{\frac{2\pi i}{3}}\,,\qquad \tau_{ij}=\delta_{ij}e^{\frac{2\pi i}{3}}\,. \tag{5.4}$$

The supersymmetry is broken to $\mathcal{N}=1$ in the resulting vacuum.

[3] Note that there are no Majorana Weyl representations of SO(10).

We are interested in possible non-perturbative corrections to the superpotential that can arise in this $\mathcal{N}=1$ theory. Such corrections could arise due to euclidean D3 branes wrapping divisor in T^6/Z_2. A correction to the superpotential requires two fermionic zero modes, no more or less, in the world volume theory of the euclidean D3 brane. Without flux there are 16 fermionic zero modes. This is too many (the sixteen zero modes follow from the $\mathcal{N}=4$ supersymmetry, present without flux, which also precludes a correction to the superpotential). With flux we will see below that only four zero modes survive. This is fewer in number, but still too many for a non-perturbative contribution to the superpotential.

A general divisor takes the form, $n_i Z^i = c$, where n_i are integers and c is a constant. We first examine the divisor $Z^3 = c$ below. In this case the D3 wraps the x^1, x^2, y^1, y^2, directions with x^3, y^3, held constant. For now we also exclude the special values, $c = 0, 1/2i, 1/2, 1/2 + 1/2i$. At these special values the Z_2 orientifold symmetry relates points on the divisor to each other. This complicates the analysis somewhat. Towards the end of the section we will consider the more general divisor. Using the symmetries of the problem we will find that the analysis can be mapped to the case when $Z^3 = c$, thus resulting in the same number of zero modes.

We ignore the five-form flux also we set $F - B$ on the world volume to zero.[4] The fermion bilinear term of the action then takes the form

$$S(\Theta^2) = -\mu_3 \int d^4\zeta e^{-\phi}\sqrt{\det\, g}\left(\frac{1}{48}\bar{\Theta}\Gamma^{mnp}\Theta H_{mnp} - \frac{1}{16}\bar{\Theta}\Gamma_{\tilde{i}pq}\Theta H^{\tilde{i}pq}\right) + \\ + \frac{\mu_3}{32}\int d^4\zeta \epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\left(\frac{1}{36}\bar{\Theta}\Gamma_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}{}^{pqr}\Theta F'_{pqr} + \bar{\Theta}\Gamma_{\tilde{i}\tilde{j}}{}^{p}\Theta F'_{\tilde{k}\tilde{l}p}\right). \tag{5.5}$$

In this equation Θ is a Weyl fermion of SO(10) but $\bar{\Theta}$ actually stands for $\Theta^T\gamma^0$, as was explained above. We see that the flux gives rise to mass terms for the fermion Θ.

The flux, eq. (5.3) does not fix all the Kahler moduli. With the choice,

$$ds^2 = \sum_{a=1}^{3} r_a^2 dz^a d\bar{z}^a \tag{5.6}$$

it is easy to see that the Kahler moduli r_a^2, contribute an overall multiplicative factor to the mass terms above. Since our main goal is to count the zero modes here, we will work with $r_a = 1$ below.

Now let us write the mass terms above as,

$$S(\Theta^2) = \frac{\mu_3}{8}\int d^4\xi\sqrt{\det g}\bar{\Theta}M\Theta\,, \tag{5.7}$$

where the matrix M is determined by the flux. We are interested in the number of zero modes of M.

[4]For the flux, eq. (5.3), we can work in a gauge where the two-form RR gauge potential $C_{(2)}$ has non-zero components, $C_{(2)x^1x^3}$, $C_{(2)y^1y^3}$. Since the brane extends along, x^1, x^2, y^1, y^2, there is then no source term for $F - B$ on the world volume and setting it to zero is consistent with the equations of motion for the world volume gauge field.

As we discuss in appendix A.6, it is convenient to work in the following basis for the analysis. Label the 16 components of Θ as $|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle$ where $\epsilon_i = \pm 1, i = 1, \ldots, 3$ refer to the eigenvalues of $\Gamma^{\hat{x}^j \hat{y}^j}$ respectively.[5] E.g.,

$$\Gamma^{\hat{x}^1 \hat{y}^1} |\epsilon_1, \epsilon_2, \epsilon_3, a\rangle = i\epsilon_1 |\epsilon_1, \epsilon_2, \epsilon_3, a\rangle \,. \tag{5.8}$$

And $a = \pm 1$ is an extra label (The SO(10) rotation group has a SO(4) $\times$ SO(6) subgroup where the SO(6) refers to the compactified directions. The label a refers to the SO(4), it takes only two values because the ten dimensional chirality is fixed.). Now it is easy to see that M acts on the state, $|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle$, as follows,

$$M|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle = \left(\frac{2}{\sqrt{3}}\right)^3 m \Gamma^{\hat{y}^1 \hat{y}^2 \hat{y}^3} |\epsilon_1, \epsilon_2, \epsilon_3, a\rangle \,, \tag{5.9}$$

where,

$$\begin{aligned} m = \frac{1}{2}\left(\sqrt{3} + \epsilon_1\epsilon_2\right) \Big\{ & e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2+\epsilon_3)} - 2 - e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2)} - e^{-\frac{i\pi}{3}(\epsilon_2+\epsilon_3)} - \\ & - e^{-\frac{i\pi}{3}(\epsilon_3+\epsilon_1)} + e^{-\frac{i\pi}{3}\epsilon_1} + e^{-\frac{i\pi}{3}\epsilon_2} + e^{-\frac{i\pi}{3}\epsilon_3} \Big\} + \\ + \epsilon_1\epsilon_2 \left(1 + e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2+\epsilon_3)}\right) . & \end{aligned} \tag{5.10}$$

Our notation for the matrix $\Gamma^{\hat{y}^1 \hat{y}^2 \hat{y}^3}$ is defined in appendix A.6. We note here that $(\Gamma^{\hat{y}^1 \hat{y}^2 \hat{y}^3})^2 = -1$ and thus $\Gamma^{\hat{y}^1 \hat{y}^2 \hat{y}^3}|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle$ cannot vanish. This means that the rhs of eq. (5.9) can vanish only if m vanishes.

It is easy to see from eq. (5.10) that this happens when when $\epsilon_1 = \epsilon_2 = \epsilon_3 = \pm 1$. As discussed in appendix A.6, this is the only choice of $\epsilon_1, \epsilon_2, \epsilon_3$ for which m vanishes. Since a in addition can take values ± 1, we get four zero modes in all.

This example illustrates the fact that fluxes can lift zero modes, although in this case we see that the remaining number is still too large for a contribution to the superpotential.

5.3 Discussion

The analysis of zero modes in [3] cannot be directly applied to the example above, since the M-theory lift of the T^6/Z_2 orientifold is a space of reduced holonomy. Still, an analogous index can be defined in this example. The U(1) symmetry here corresponds to rotations in the plane formed by the x^3, y^3 directions. The U(1) charge of a zero mode is therefore simply ϵ_3. The graded index is then,

$$\chi \equiv \sum (-1)^{\epsilon_3} \,, \tag{5.11}$$

where the sum is over all the fermionic zero-modes. In the absence of flux, there are 8 zero modes with $\epsilon_3 = +1$ and 8 with $\epsilon_3 = -1$ so this index vanishes. In the presence of flux, there are 2 zero modes with charge ± 1 each so again the index vanishes.

We see from eq. (5.3) that the three-form fluxes H, F have two legs along the divisor and one normal to it, and so break the U(1) symmetry. In the basis above, the U(1) symmetry

[5] Here the '$\hat{}$' indicates that we are in the vierbein basis.

relates the two states $\{|\epsilon_1, \epsilon_2, \pm\epsilon_3, a\rangle\}$ to each other. And we see from appendix A.6, that these states do get different masses, in accordance with the breaking of the symmetry.

The flux eq. (5.3) is invariant under the Z_2 orientifold symmetry as required. In this example there are additional Z_2 symmetries as well, however. These involve inverting the coordinates of only one of the torii, while keeping the other torii fixed. E.g., $(x^1, y^1) \to -(x^1, y^1)$ while keeping the other coordinates fixed. The two-forms, C_2, B_2 have odd intrinsic parity under these Z_2's (as they do under the Z_2 orientifold symmetry) and thus the flux eq. (5.3) is invaraint under them. As a result of these additional symmetries, the zero modes can only be lifted in pairs, with the states $\{|\epsilon_1\epsilon_2, \epsilon_3, a\rangle, |-\epsilon_1, -\epsilon_2, -\epsilon_3, a\rangle\}$ having the same mass.[6] This explains why the index does not change after incorporating the flux. In the more generic case of a Calabi-Yau space with flux, such additional symmetries would be absent while the feature that the flux breaks the U(1) symmetry continues to be true, so one expects that the index can change after incorporating flux. Evidence for this was already found in [4] for the case of M theory on $K3 \times K3$. There it was argued that for a divisor of the form $K3 \times P^1$, zero modes coming from $h(2,0)$ of the divisor, which have the same U(1) charge, pair up amongst themselves and get heavy.

After turning on the flux, eq. (5.3), $\mathcal{N} = 1$ supersymmetry is left unbroken in the resulting vacuum. The D3 brane wrapping the divisor breaks some of these supersymmetries, and this gives rise to some zero modes in the D3 brane world volume theory. It would be helpful to know which of the four zero modes we have found above are related to the breaking of supersymmetry. We have not analysed this question in detail and leave it for the future. Let us note in passing here that the conditions for supersymmetry imposed by the D3 brane are independent of the three-form flux. In the absence of flux, half the supersymmetries are left unbroken by the D3 brane wrapping a divisor, this suggests that two of the four supersymmetries are broken by the D3 brane, and two of the four zero modes are due to this partial breaking of supersymmetry.[7]

We have focussed on a specific divisor above, $Z^3 = c$. The case when Z^3 is replaced by Z^1, Z^2 gives the same zero-mode count due to the symmetries of the flux, eq. (5.3). Also in the discussion above we have excluded some special values, $c = 0, 1/2i, 1/2, 1/2(1+i)$. The divisors for these values of c are special. The Z_2 orientifold symmetry relates points on the divisor to each other in these cases so the divisors are "half-cycles". Starting with a situation where the brane is away from one of these special values of c we can continuously move it to the special values. The brane and its image under the Z_2 orientifold symmetry come together then. Since the brane can be moved continuously in this way we do not expect the number of zero modes to jump. A more interesting possibility is that of a brane without its image wraping one of these special divisors. This would be the analogue of a fractional brane. It is tempting to speculate that the Z_2 orientifold symmetry acts on the fermions in this case and halves their number, resulting in two zero modes - just the correct number for an instanton correction. Partial evidence for this comes from anomaly considerations. Since the brane wraps a half-cycle its action depends only fractionally on

[6]More correctly we are interested in the eigenmodes of $M^\dagger M$. Both $\{|\epsilon_1\epsilon_2, \epsilon_3, a\rangle, |-\epsilon_1, -\epsilon_2, -\epsilon_3, a\rangle\}$ are eigenvectors of this operator with the same eigenvalues.

[7]We are grateful to Rudra Jena for a discussion in this regard.

the Kahler modulus govering the volume of the four cycle and also depends only fractionally on the axionic partner of this Kahler modulus. This suggests that the number of zero modes is also halved. We have not fully explored this interesting case yet and hope to return to it in the future.

A more general divisor has the form, $n_i Z^i = c$. As discussed towards the end of appendix A.6, upto an overall rescaling of the mass matrix, the analysis for the more general divisor can be mapped to the case where one of the three coordinates, Z^1, Z^2 or Z^3 is a constant. Thus the discussion above applies and we learn (again with the possible exception of some special values of c) that for the case of a more general divisor as well there are four fermion zero modes.

Finally, the zero modes we have found are constant spinors which are zero modes of the mass matrix, eq. (5.7). They are therefore zero modes of the Dirac operator,

$$\not{D}\Theta + \frac{1}{4}M\Theta = 0\,, \tag{5.12}$$

since both term above vanish seperately acting on the zero modes. One could ask if there are additional non-constant zero modes of the Dirac operator.[8] Under a rescaling of the volume of the internal space, $g_{mn} \to \lambda^2 g_{mn}$ (where now m, n take values only over the six internal space directions) one finds that $\not{D} \to \frac{1}{\lambda}\not{D}$ while $M \to \frac{1}{\lambda^3}M$. Thus at large volume the first term, $\not{D}\Theta$, is much more important and our approximation of starting with constant spinors and seeking zero modes of M amongst them is justified. Additional non-constant zero modes of the Dirac operator eq. (5.12) can be found in this example, but in agreement with the argument just mentioned they always occur at a volume of order the string scale. For such small volumes the α' corrections are important and the analysis is not trustworthy.[9]

6. Conclusions

In this paper we have used the method of gauge completion and determined the fermion bilinear terms in the world volume action of a D3 brane in the presence of background flux. Our results are summarised in eq. (3.7) and eq. (3.8). These results have been previously obtained by Marolf, Martucci and Silva using somewhat different methods.

The fermion bilinear terms are of interest in calculating instanton corrections to the superpotential in flux compactifications. They are also of interest in determining soft susy breaking terms that can arise in flux compactifications.

For a euclidean D3 brane wrapping a divisor in a six dimensional compactification these results explicitly show that the U(1) symmetry of rotations normal to the divisor is broken in the presence of three-form flux. In an explcit example of a T^6/Z_2 compactification with three-form flux we have calculated the fermion mass terms and shown that many zero modes are lifted due to the flux.

[8] We thank Renata Kallosh for a discussion of this issue.

[9] In other examples of a Calabi-Yau space with large orientifold charge the flux can be bigger and it might be possible to have the two terms in eq. (5.12) comparable to each other when the volume is bigger than the string scale.

There are several directions of future work. One would like a better understanding of supersymmetry in this context. This is connected to the number of zero modes in the world volume of the D3 brane. More generally, one would like to use our results to calculate the instanton corrections in situations where they can arise. Even in the simple example studied here, of a T^6/Z_2 compactification, our analysis is not complete and the case when the $D3$ brane wraps a half-cycle needs to be understood better.

We hope to return to these questions in the future.

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A. Computational details

Our conventions are as follows.

Superspace coordinates are denoted by $Z^M = (x^m, \theta^\mu)$, which stand for the bosonic and fermionic components respectively. Curved space indices are given by $\{M, N, \ldots\} = \{m, n, \ldots, \mu, \nu, \ldots\}$ where (m, n) denote Bosonic indices and (μ, ν) fermionic indices. Tangent space indices are given by $\{A, B, \ldots\} = \{a, b, \ldots, \alpha, \beta, \ldots\}$, with (a, b) denoting bosonic and (α, β) fermionic indices.

We will use real 16 component Majorana-Weyl spinors, a convenient basis of gamma matrices is given in eq.(4.3.48), [33], in which Γ^0 is antisymmetric and the remaining gamma matrices, Γ^i, are symmetric. Since there are two 16 component Majorana-Weyl spinors worth of supersymmetries in the IIB theory our spinors will carry an extra SO(2) index. The spinor indices α, β, should be viewed as composite indices standing for the tensor product of a Majorana-Weyl index and this additional SO(2) index. In the formulae below the gamma matrices will act on the Majorana Weyl index while the Pauli spin matrices, $\sigma^1, \sigma^2, \sigma^3$, will act on the SO(2) index.

Throughout this paper, we denote antisymmetrisation with unit weight by a square bracket. For example, the antisymmetrised product of an antisymmetric rank-two tensor A_{mn} with a rank one tensor B_p is,

$$A_{[mn}B_{p]} = \frac{1}{3}[A_{mn}B_p - A_{pn}B_m - A_{mp}B_n]\,. \tag{A.1}$$

There are 3 distinct terms which appear on the rhs as shown. To make it of unit weight we divide by the number of distinct terms, which accounts for the prefactor $\frac{1}{3}$. Finally, $\Gamma_{m_1 \cdots m_n} = \Gamma_{[m_1} \cdots \Gamma_{m_n]}$, will denote the antisymmetrised product of n Gamma matrices.

A.1 Supersymmetry transformations

With these conventions, the supersymmetric transformation rules in the string frame [34] are given by[10]

$$
\begin{aligned}
\delta\lambda &= \frac{1}{2}\Gamma^m\partial_m\phi\epsilon - \frac{1}{24}\Gamma^{mnp}H_{mnp}\sigma^3\epsilon - \frac{1}{2}e^\phi\Gamma^m F_m(i\sigma^2)\epsilon - \frac{1}{24}e^\phi\Gamma^{mnp}F'_{mnp}\sigma^1\epsilon \\
\delta\psi_m &= D_m\epsilon + \frac{1}{8}e^\phi\Gamma^p\Gamma_m F_p(i\sigma^2)\epsilon - \frac{1}{8}\Gamma^{pq}H_{mpq}\sigma^3\epsilon + \\
&\quad + \frac{1}{48}e^\phi\Gamma^{pqr}\Gamma_m F'_{pqr}\sigma^1\epsilon + \frac{1}{16\cdot 5!}e^\phi\Gamma^{pqrst}\Gamma_m F'_{pqrst}(i\sigma^2)\epsilon \\
\delta\phi &= \bar\epsilon\lambda \\
\delta C &= e^{-\phi}\bar\epsilon\sigma^1\lambda \\
\delta e_m{}^a &= \bar\epsilon\Gamma^a\psi_m \\
\delta B_{mn} &= \bar\epsilon\sigma^3\left(\Gamma_m\psi_n - \Gamma_n\psi_m\right) \\
\delta C_{mn} &= e^{-\phi}\bar\epsilon\sigma^1\left(\Gamma_{mn}\lambda - 2\Gamma_{[m}\psi_{n]}\right) + C\delta B_{mn} \\
\delta C_{mnpq} &= e^{-\phi}\bar\epsilon(i\sigma^2)\left(\Gamma_{mnpq}\lambda - 4\Gamma_{[mnp}\psi_{q]}\right) + 6C_{[mn}\delta B_{pq]}\,.
\end{aligned} \tag{A.2}
$$

Here F'_3 and F'_5 are the gauge invariant RR field strengths

$$F'_3 = dC_2 - CH_3\,, \qquad F'_5 = dC_4 - C_2\wedge H_3\,. \tag{A.3}$$

Using these supersymmetry transformations, in the following sections we will compute the the expansion of the superfields $\hat{C}_{MN}, \hat{C}_{MNPQ}$ and $\hat{e}^A_M$, up to $O(\theta^2)$, in terms of the component fields.

A.2 Calculation for $\hat{C}_{MN}$

Following similar steps for the calculation of $\hat{B}_{MN}$ in section 2.2, here we will carry out the expansion of $\hat{C}_{MN}$ to $O(\theta^2)$. For this purpose, we must supply the superspace gauge transformation $\Sigma^{(c)}_M$ to $O(\theta)$, in addition to the super diffeomorphism (2.13).

Let us first evaluate the commutator of two supersymmetry transformations (with parameters ϵ^1, ϵ^2) on the field C_{mn}. Using eq. (A.2) for the supersymmetry transformation of C_{mn} we find that

$$\delta_1\delta_2 C_{mn} = e^{-\phi}\bar\epsilon_2\left(\Gamma_{mn}\delta_1\lambda - 2\Gamma_{[m}\delta_1\psi_{n]}\right) + C\delta_1\delta_2 B_{mn}\,. \tag{A.4}$$

Using eq. (A.2) once more, it is straightforward to see that the commutator can be expressed as a diffeomorphism (with the parameter ξ^m as given by eq. (2.13)), and a gauge transformation $\xi^{(c)}_m$. In other words

$$(\delta_1\delta_2 - \delta_2\delta_1)C_{mn} = \xi^p\partial_p C_{mn} + \partial_m\xi^p C_{pn} - \partial_n\xi^p C_{pm} + \partial_m\xi^{(c)}_n - \partial_n\xi^{(c)}_m\,, \tag{A.5}$$

with the gauge transformation parameter

$$\xi^{(c)}_m = \xi^n C_{mn} + e^{-\phi}\bar\epsilon_2\sigma^1\Gamma_m\epsilon_1 - C\bar\epsilon_2\sigma^3\Gamma_m\epsilon_1\,. \tag{A.6}$$

[10]Note that here our normalization for λ is different from ref. [34].

In order to find the gauge transformation parameter, we have to compare eq. (A.5) with the commutator derived in the superspace formalism. It is easy to see that

$$(\delta_1\delta_2 - \delta_2\delta_1)\hat{C}_{MN} = \left(\partial_M \Sigma_N^{12(c)} - (-1)^{MN}\partial_N \Sigma_N^{12(c)}\right) + \cdots , \tag{A.7}$$

where the dots denote terms arising due to superdiffeomorphism. The superspace gauge transformation parameter $\Sigma_M^{12(c)}$ is given by

$$\Sigma_M^{12(c)} = \left(\Sigma_2^P \partial_P \Sigma_M^{1(c)} + \partial_M \Sigma_2^P \Sigma_P^{1(c)}\right) - (1 \leftrightarrow 2) . \tag{A.8}$$

The commutator on component field C_{mn} will agree with the commutator on the superfield $\hat{C}_{mn}$ if the gauge transformation parameter $\Sigma_m^{(c)}$ takes the value

$$\Sigma_m^{(c)} = \frac{1}{2}\bar{\theta}\Gamma^n \epsilon C_{mn} + \frac{1}{2}\bar{\theta}\left(e^{-\phi}\sigma^1 - C\sigma^3\right)\Gamma_m \epsilon . \tag{A.9}$$

The component fields $C_{m\mu}$ and $C_{\mu\nu}$ are both zero to leading order and hence the commutator of the two susy on them also vanishes. From this it is easy to see that the component $\Sigma_\mu^{(c)}$ vanishes for the case when the the space time fermion backgrounds are set to zero.

Now let us compute the first order expansion for $\hat{C}_{mn}$. Comparing the susy transformation for C_{mn} from eq. (A.2) with the superfield result $\delta\hat{C}_{mn} = \epsilon^\alpha \partial_\alpha \hat{C}_{mn}$, we find

$$\hat{C}_{mn} = C_{mn} + e^{-\phi}\bar{\theta}\sigma^1\left(\Gamma_{mn}\lambda - 2\Gamma_{[m}\psi_{n]}\right) + 2C\bar{\theta}\Gamma_{[m}\psi_{n]} . \tag{A.10}$$

The expression for $\hat{C}_{m\mu}$ can similarly be derived. Using the expression for the gauge transformation (A.9) the superspace variation for $\hat{C}_{m\mu}$ can be written as

$$\delta\hat{C}_{m\mu} = \epsilon^\alpha \partial_\alpha \hat{C}_{m\mu} - \frac{1}{2}e^{-\phi}\left(\bar{\epsilon}\sigma^1\Gamma_m\right)_\mu + \frac{1}{2}C\left(\bar{\epsilon}\sigma^3\Gamma_m\right)_\mu . \tag{A.11}$$

Since the component field susy transformation $\delta C_{m\mu} = 0$, the r.h.s. of eq. (A.11) has to be equated to zero. This gives the expression

$$\hat{C}_{m\mu} = \frac{1}{2}e^{-\phi}\left(\bar{\theta}\sigma^1\Gamma_m\right)_\mu - \frac{1}{2}C\left(\bar{\theta}\sigma^3\Gamma_m\right)_\mu . \tag{A.12}$$

With the help of this equation and the gauge transformation (A.9), we can write down the variation of the superfield $\hat{C}_{mn}$ up to $O(\theta)$.

$$\begin{aligned}\delta\hat{C}_{mn} &= \epsilon^\alpha\partial_\alpha\hat{C}_{mn} - 2\bar{\theta}\left(e^{-\phi}\sigma^1 - C\sigma^3\right)\Gamma_{[m}\partial_{n]}\epsilon + \bar{\theta}\left(e^{-\phi}\sigma^1 - C\sigma^3\right)\Gamma^a\epsilon\partial_{[m}e_{n]a} - \\ &- \frac{1}{2}\bar{\epsilon}\Gamma^q\theta F_{qmn} + \left(e^{-\phi}\bar{\epsilon}\sigma^1\Gamma_{[m}\theta\partial_{n]}\phi + \bar{\epsilon}\sigma^3\Gamma_{[m}\theta\partial_{n]}C\right) .\end{aligned} \tag{A.13}$$

On the other hand, we can use eq. (A.10) for $\hat{C}_{mn}$ to arrive at

$$\delta\hat{C}_{mn} = \delta C_{mn} + e^{-\phi}\bar{\theta}\sigma^1\left(\Gamma_{mn}\delta\lambda - 2\Gamma_{[m}\delta\psi_{n]}\right) + 2C\bar{\theta}\Gamma_{[m}\delta\psi_{n]} . \tag{A.14}$$

These two variations must be the same. When we plug in the susy transformations for ψ_m and λ from eq. (A.2), we find that they will match up only when $\hat{C}_{mn}$ has the following expression to second order in θ:

$$\hat{C}_{mn} = C_{mn} + e^{-\phi}\bar{\theta}\sigma^1\left(\Gamma_{mn}\lambda - 2\Gamma_{[m}\psi_{n]}\right) + 2C\bar{\theta}\Gamma_{[m}\psi_{n]} + \frac{1}{4}e^{-\phi}\bar{\theta}\sigma^1\Gamma_{mnp}\theta\partial^p\phi +$$

$$
\begin{aligned}
&+\frac{1}{8}\bar{\theta}\left(e^{-\phi}\sigma^1 - C\sigma^3\right)\left(\Gamma_m{}^{ab}\omega_{nab} - \Gamma_n{}^{ab}\omega_{mab}\right)\theta + \frac{1}{8}\bar{\theta}\left(\sigma^3 - e^{\phi}C\sigma^1\right)\Gamma_{mnp}\theta F^p +\\
&+\frac{1}{8}e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma^p\theta H_{mnp} + \frac{1}{48}e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{mn}{}^{pqr}\theta H_{pqr} - \frac{1}{8}C\bar{\theta}\Gamma_{[m}{}^{pq}H_{n]pq}\theta -\\
&-\frac{1}{8}\bar{\theta}\Gamma_{[m}{}^{pq}F'_{n]pq}\theta - \frac{1}{8}Ce^{\phi}\bar{\theta}(i\sigma^2)\Gamma^p\theta F'_{mnp} - \frac{1}{48}Ce^{\phi}\bar{\theta}(i\sigma^2)\Gamma_{mn}{}^{pqr}\theta F'_{pqr} -\\
&-\frac{1}{16\cdot 5!}\bar{\theta}\left(\sigma^3 + Ce^{\phi}\sigma^1\right)\left(\Gamma_{mn}{}^{pqrst}F'_{pqrst} + 20\Gamma^{pqr}F'_{mnpqr}\right)\theta\,. \qquad \text{(A.15)}
\end{aligned}
$$

A.3 Calculation for $\hat{C}_{MNPQ}$

Let us now turn to $\hat{C}_{MNPQ}$. The calculation is pretty much the same as the previous ones for $\hat{B}_{MN}$ and $\hat{C}_{MN}$. We will first evaluate the gauge transformation parameter Σ_{MNP} from the commutator of two susy transformation on the component field C_4 and then use this information to derive the $O(\theta^2)$ expression for the superfield $\hat{C}_4$. From eq. (A.2) we find

$$
\delta_1\delta_2 C_{mnpq} = e^{-\phi}\bar{\epsilon}_2(i\sigma^2)\left(\Gamma_{mnpq}\delta_1\lambda - 4\Gamma_{[mnp}\delta_1\psi_{q]}\right) + 12\bar{\epsilon}_2\sigma^3 C_{[mn}\Gamma_p\delta_1\psi_{q]}\,. \qquad \text{(A.16)}
$$

After some straightforward calculation, the commutator of two susy transformations can be written as

$$
\begin{aligned}
(\delta_1\delta_2 - \delta_2\delta_1)C_{mnpq} = \bar{\epsilon}_2\Big(&-4e^{-\phi}(i\sigma^2)\partial_{[m}\phi\Gamma_{npq]} + \Gamma^a F'_{amnpq} + 4\sigma^3\Gamma_{[m}F'_{npq]} +\\
&+ 4\sigma^1\Gamma_{[m}H_{npq]} + 6C_{[mn}\Gamma^a H_{pq]a}\Big)\epsilon_1\,. \qquad \text{(A.17)}
\end{aligned}
$$

This is equal to a diffeomorphism with diffeomorphism parameter ξ^m as given in eq. (2.11), and a gauge transformation

$$
d\xi_3 + d(H_3 \wedge \xi^{(c)})\,,
$$

with the gauge transformation parameter ξ_3 having the expression

$$
\xi_{mnp} = \xi^q C_{mnpq} + e^{-\phi}\bar{\epsilon}_2(i\sigma^2)\Gamma_{mnp}\epsilon_1 - 3C_{[mn}\bar{\epsilon}_2\sigma^3\Gamma_{p]}\epsilon_1\,. \qquad \text{(A.18)}
$$

Now we can evaluate the commutator on the super field $\hat{C}_4$,

$$
(\delta_1\delta_2 - \delta_2\delta_1)\hat{C}_4 = d\left(\Sigma_3^{12}\right) + \cdots\,. \qquad \text{(A.19)}
$$

Here again the dots denote the superdiffeomorphisms. The gauge transformation parameter Σ_3^{12} can be written in terms of components as

$$
\Sigma^{12}_{MNP} = \left[\left(\Sigma_2^Q\partial_Q\Sigma_{1MNP} + 3\partial_{[M}\Sigma_2^Q\Sigma_{1NP]Q}\right) - (1\leftrightarrow 2)\right]. \qquad \text{(A.20)}
$$

Comparing the two commutators we can easily solve for the gauge transformation parameter Σ_{mnp} to obtain

$$
\Sigma_{mnp} = \frac{1}{2}\bar{\theta}\Gamma^q\epsilon C_{mnpq} + \frac{1}{2}e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{mnp}\epsilon - \frac{3}{2}C_{[mn}\bar{\theta}\sigma^3\Gamma_{p]}\epsilon\,. \qquad \text{(A.21)}
$$

All the remaining components of Σ_{MNP} will be zero.

We now need to evaluate the expressions for $\hat{C}_{mnpq}$ and $\hat{C}_{\mu mnp}$ to $O(\theta)$. It is easy to see that the variation $\delta\hat{C}_{mnpq} = \epsilon^\alpha\partial_\alpha\hat{C}_{mnpq}$, and the susy transformation for C_{mnpq} from eq. (A.2) gives

$$\hat{C}_{mnpq} = C_{mnpq} + e^{-\phi}\bar{\theta}(i\sigma^2)\left\{\Gamma_{mnpq}\lambda - 4\Gamma_{[mnp}\psi_{q]}\right\} + 12\bar{\theta}\sigma^3 C_{[mn}\Gamma_p\psi_{q]}\,. \qquad \text{(A.22)}$$

Using the expression for the gauge transformation (A.21), we can obtain the variation $\delta\hat{C}_{\mu mnp}$. Since $C_{\mu mnp}$ vanishes to leading order, this variation has to be set to zero. As a result, we get

$$\hat{C}_{\mu mnp} = -\frac{1}{2}e^{-\phi}\left(\bar{\theta}(i\sigma^2)\Gamma_{mnp}\right)_\mu + \frac{3}{2}\left(C_{[mn}\bar{\theta}\sigma^3\Gamma_{p]}\right)_\mu\,. \qquad \text{(A.23)}$$

It is easy to see that all the remaining components of $\hat{C}_{MNPQ}$ vanishes. Now we are ready to execute the second order results.Using the above expression for $\hat{C}_{\mu mnp}$ and the expression for the gauge transformation parameter from eq. (A.21) we find the variation of $\hat{C}_{mnpq}$ to be of the form

$$\begin{aligned}\delta\hat{C}_{mnpq} &= \epsilon^\alpha\partial_\alpha\hat{C}_{mnpq} + \frac{1}{2}\bar{\theta}\Gamma^a\epsilon F_{amnpq} - 4H_{[mnp}\Sigma^{(c)}_{q]} + 4\partial_{[m}\epsilon^\alpha\hat{C}_{\alpha npq]} + \\ &\quad +2\partial_{[m}\left(e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{npq]}\epsilon\right) - 6\partial_{[m}\left(C_{np}\bar{\theta}\sigma^3\Gamma_{q]}\epsilon\right). \end{aligned} \qquad \text{(A.24)}$$

After substituting the expression for $\Sigma^{(c)}_m$ and making some rearrangement we get

$$\begin{aligned}\delta\hat{C}_{mnpq} &= \epsilon^\alpha\partial_\alpha\hat{C}_{mnpq} + \frac{1}{2}\bar{\theta}\Gamma^a\epsilon(F_{amnpq} - 4H_{[mnp}C_{q]a}) + \\ &\quad +2\bar{\theta}e^{-\phi}\sigma^1\Gamma_{[m}\epsilon H_{npq]} + 4\partial_{[m}\epsilon^\alpha\hat{C}_{\alpha npq]} + \\ &\quad +2\bar{\theta}\left\{\partial_{[m}\left(e^{-\phi}(i\sigma^2)\Gamma_{npq]}\epsilon\right) - 3C_{[np}\partial_m\left(\sigma^3\Gamma_{q]}\epsilon\right) - F'_{[mnp}\sigma^3\Gamma_{q]}\epsilon\right\}. \end{aligned} \qquad \text{(A.25)}$$

We can also obtain the variation from eq. (A.22) for the expansion of $\hat{C}_{mnpq}$ to $O(\theta)$:

$$\delta\hat{C}_{mnpq} = \delta C_{mnpq} + e^{-\phi}\bar{\theta}(i\sigma^2)\left\{\Gamma_{mnpq}\delta\lambda - 3\Gamma_{[mnp}\delta\psi_{q]}\right\} + 12\bar{\theta}\sigma^3 C_{[mn}\Gamma_p\delta\psi_{q]}\,. \qquad \text{(A.26)}$$

These two expressions must agree. This can be used to solve for $\hat{C}_{mnpq}$ to second order in θ to obtain

$$\begin{aligned}\hat{C}_{mnpq} &= C_{mnpq} + e^{-\phi}\bar{\theta}(i\sigma^2)\left\{\Gamma_{mnpq}\lambda - 4\Gamma_{[mnp}\psi_{q]}\right\} + 12\bar{\theta}\sigma^3 C_{[mn}\Gamma_p\psi_{q]} + \\ &\quad +\frac{1}{2}e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{ab[mnp}\omega_{q]}{}^{ab}\theta + 3e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{[p}\omega_{qmn]}\theta + \frac{1}{4}e^{-\phi}\bar{\theta}(i\sigma^2)\Gamma_{mnpq}{}^s\partial_s\phi\theta + \\ &\quad +\frac{1}{48}e^{-\phi}\bar{\theta}\sigma^1\Gamma_{mnpq}{}^{stu}H_{stu}\theta + \frac{1}{48}\bar{\theta}\sigma^3\Gamma_{mnpq}{}^{stu}F'_{stu}\theta + \frac{1}{2}\bar{\theta}\Gamma_{[mnp}F_{q]}\theta + \\ &\quad +\frac{3}{4}\bar{\theta}\sigma^3\Gamma_{[mn}{}^sF'_{pq]s}\theta + \frac{3}{4}e^{-\phi}\bar{\theta}\sigma^1\Gamma_{[mn}{}^sH_{pq]s}\theta - \frac{1}{96}\bar{\theta}\Gamma_{[mnp}{}^{stuv}F'_{q]stuv}\theta - \\ &\quad -\frac{1}{8}\bar{\theta}\Gamma_{[m}{}^{st}F'_{npq]st}\theta - \frac{3}{2}\bar{\theta}\sigma^3 C_{[mn}\Gamma_p{}^{ab}\omega_{q]ab}\theta - \\ &\quad -\frac{3}{4}e^\phi C_{[mn}\bar{\theta}\Bigg(\sigma^1\Gamma_{pq]}{}^sF_s + e^{-\phi}\Gamma_p{}^{st}H_{q]st} + i\sigma^2\left\{\Gamma^sF'_{pq]s} + \frac{1}{6}\Gamma_{pq]}{}^{stu}F'_{stu}\right\} + \\ &\qquad\qquad + \frac{1}{12}\sigma^1\left\{\Gamma^{stu}F'_{pq]stu} + \frac{1}{20}\Gamma_{pq]}{}^{stuvw}F'_{stuvw}\right\}\Bigg)\theta\,. \end{aligned} \qquad \text{(A.27)}$$

A.4 The supervierbein

Finally we come to the computation of the vierbeins. A similar calculation can be performed in this case also. Note that in addition to the superdiffeomorphism, here we have to consider the (super) local Lorentz transformation (2.3). Let us first compute vierbeins to $O(\theta)$. Equating $\delta\hat{e}^a_m = \epsilon^\alpha\partial_\alpha\hat{e}^a_m$ with $\delta e_m{}^a = \bar{\epsilon}\Gamma^a\psi_m$ we find

$$\hat{e}^a_m = e_m{}^a + \bar{\theta}\Gamma^a\psi_m\,. \tag{A.28}$$

Similarly we can compute $\hat{e}^a_\mu$ to $O(\theta)$. using the value of Σ^m from eq. (2.13) we find,

$$\hat{e}^a_\mu = -\frac{1}{2}\left(\bar{\theta}\Gamma^a\right)_\mu\,. \tag{A.29}$$

To obtain the Lorentz transformation parameter to $O(\theta)$, we need to compute the commutator of two supersymmetry transformations on the vierbein $e_m{}^a$. This can be easily computed using the susy transformations (A.2). After some simplification, we get

$$\begin{aligned}(\delta_2\delta_1-\delta_1\delta_2)\,e_m{}^a =\;& (\bar{\epsilon}_1\Gamma^a\partial_m\epsilon_2-\bar{\epsilon}_2\Gamma^a\partial_m\epsilon_1)-\bar{\epsilon}_1\Gamma_b\epsilon_2\omega_m{}^{ab}+\frac{1}{4}e^\phi\bar{\epsilon}_1\Gamma^{ap}{}_m(i\sigma^2)\epsilon_2F_p-\\ &-\frac{1}{2}\bar{\epsilon}_1\Gamma_q\sigma^3\epsilon_2H_m{}^{aq}+\frac{1}{24}e^\phi\bar{\epsilon}_1\Gamma^{apqr}{}_m\sigma^1\epsilon_2F'_{pqr}+\frac{1}{4}e^\phi\bar{\epsilon}_1\Gamma_q\sigma^1\epsilon_2F'_m{}^{aq}+\\ &+\frac{1}{8\cdot5!}e^\phi\bar{\epsilon}_1\Gamma^{apqrst}{}_m(i\sigma^2)F'_{pqrst}\epsilon_2+\frac{1}{48}e^\phi\bar{\epsilon}_1\Gamma^{pqr}(i\sigma^2)\epsilon_2F'_{mapqr}\,.\end{aligned} \tag{A.30}$$

The above equation can be written in the following simple form

$$(\delta_1\delta_2-\delta_2\delta_1)\,e_m{}^a = \xi^n\partial_ne_m{}^a+(\partial_m\xi^n)e_n{}^a+\lambda^{ab}e_{mb}\,, \tag{A.31}$$

provided the translation parameter ξ^n is given in eq. (2.11), and the rotation parameter λ^{ab} has the expression

$$\begin{aligned}\lambda^{ab} =\;& -\xi^n\omega_n{}^{ab}+\frac{1}{2}\bar{\epsilon}_2\Gamma_p\sigma^3\epsilon_1H^{abp}-\\ &-\frac{1}{4}e^\phi\bar{\epsilon}_2\Big\{\Gamma^{abp}(i\sigma^2)F_p+\frac{1}{6}\Gamma^{abpqr}\sigma^1F'_{pqr}+\Gamma_p\sigma^1F'^{abp}+\\ &\qquad+\frac{1}{2\cdot5!}\Gamma^{abpqrst}(i\sigma^2)F'_{pqrst}+\frac{1}{12}\Gamma_{pqr}(i\sigma^2)F'^{abpqr}\Big\}\epsilon_1\,.\end{aligned} \tag{A.32}$$

In deriving (A.31) we have used the following identity obeyed by the spin connection and the vierbein

$$e_{nb}\omega_m{}^{ab} = e_{mb}\omega_n{}^{ab}+(\partial_me_n{}^a-\partial_ne_m{}^a)\,. \tag{A.33}$$

On the other hand, one can apply the commutator directly on the super vierbein as given in eq. (A.28). This will be consistent with eq. (A.31) if the parameter Λ^{ab} takes the form

$$\begin{aligned}\Lambda^{ab}(\epsilon) =\;& -\frac{1}{2}\bar{\theta}\Gamma^n\epsilon\omega_n{}^{ab}+\frac{1}{4}\bar{\theta}\Gamma_p\sigma^3\epsilon H^{abp}-\\ &-\frac{1}{8}e^\phi\bar{\theta}\bigg(\Gamma^{abp}(i\sigma^2)F_p+\frac{1}{6}\Gamma^{abpqr}\sigma^1F'_{pqr}+\Gamma_p\sigma^1F'^{abp}+\\ &\qquad+\frac{1}{2\cdot5!}e^\phi\Gamma^{abpqrst}(i\sigma^2)F'_{pqrst}+\frac{1}{12}\Gamma_{pqr}(i\sigma^2)F'^{abpqr}\bigg)\epsilon\,.\end{aligned} \tag{A.34}$$

Now we are ready to compute the $O(\theta^2)$ part of the super vierbein. Consider the variation

$$\begin{aligned} \delta\hat{e}^a_m &= \Sigma^P\partial_P\hat{e}^a_m + \partial_m\Sigma^P\hat{e}^a_P + \Lambda^{aP}\hat{e}_{mP} \\ &= \epsilon^\alpha\partial_\alpha\hat{e}^a_m + \frac{1}{2}\bar{\theta}\Gamma^n\epsilon\left(\partial_n\hat{e}^a_m - \partial_m\hat{e}^a_n\right) + \bar{\theta}\Gamma^a\partial_m\epsilon + \Lambda^{ab}\hat{e}_{mb}\,. \end{aligned} \tag{A.35}$$

It should be equated with the variation coming from eq. (A.28),

$$\delta\hat{e}^a_m = \delta e_m{}^a + \bar{\theta}\Gamma^a\delta\psi_m\,, \tag{A.36}$$

from which it follows that

$$\epsilon^\alpha\partial_\alpha\hat{e}^a_m = \bar{\theta}\Gamma^a\delta\psi'_m - \Lambda^{ab}e_{mb} - \frac{1}{2}\bar{\theta}\Gamma^n\epsilon\left(\partial_n e_m{}^a - \partial_m e_n{}^a\right). \tag{A.37}$$

Here the prime indicates the absence of $\partial_m\epsilon$ from susy variation of the gravitino. Again using the formula for the supersymmetry transformations (A.2), the above expression can easily be integrated. The super vierbein, up to $O(\theta^2)$, takes the form

$$\begin{aligned} \hat{e}^a_m &= e_m{}^a + \bar{\theta}\Gamma^a\psi_m - \frac{1}{8}\omega_{mcd}\bar{\theta}\Gamma^{acd}\theta + \frac{1}{16}e^\phi\bar{\theta}(i\sigma^2)\left(\Gamma_m F^a + \Gamma^a F_m - \delta^a_m\Gamma^p F_p\right)\theta - \\ &\quad - \frac{1}{16}\bar{\theta}\Gamma^{apq}\sigma^3\theta H_{mpq} + \frac{1}{32}e^\phi\bar{\theta}\left(\Gamma^{apq}F'_{mpq} + \Gamma_{mpq}F'^{apq}\right)\sigma^1\theta + \\ &\quad + \frac{1}{32\cdot 4!}e^\phi\bar{\theta}\left(\Gamma^{apqrs}F'_{mpqrs} + \Gamma_{mpqrs}F'^{apqrs}\right)(i\sigma^2)\theta\,. \end{aligned} \tag{A.38}$$

A.5 T-duality

It is in fact possible to obtain the $D3$ brane action, starting with $D0$ brane action and performing three T-dualities (say, along x, y, z). For simplicity, we assume the metric to be diagonal along the directions on which we perform T-duality. Also we set $B_{xi} = g_{xi} = 0$ (and similar relations for y and z directions). Here we summarize the rules for T-duality along the direction x. See [35−37, 10, 11, 38] for the T-duality rules in presence of more general background.

$$\begin{aligned} g_{xx} &= \frac{1}{j_{xx}} \\ g_{\check{i}\check{j}} &= j_{\check{i}\check{j}} \\ e^{2\phi} &= \frac{e^{2\varphi}}{j_{xx}} \\ H &= \mathcal{H} \\ F'_{n(x)} &= F'_{(n-1)} \\ F'_n &= F'_{(n+1)(x)} \\ \gamma^x &= \gamma_x \\ \gamma^{\check{i}} &= \gamma^{\check{i}}\,. \end{aligned} \tag{A.39}$$

Here we follow the notations of ref. [38]. In particular, F'_n are gauge invariant RR field strengths and also $F_{n(x)}$ denotes an $(n-1)$ form whose components are given by

$$\left[F_{n(x)}\right]_{i_1\cdots i_{n-1}} = \left[F_n\right]_{xi_1\cdots i_{n-1}}\,. \tag{A.40}$$

A.6 The mass matrix

In this section we will evaluate the fermion bilinear term due to the three form flux when the three brane wraps a divisor of T^6. Here we will use the coordinates $\{x^j, y^j\}$, $j = 1, \ldots, 3$ to parametrize the spatial directions of the torus and $\{\hat{x}^j, \hat{y}^j\}$ for the corresponding tangent space indices. Now consider the relevant part of the action as given in eq. (5.5):

$$\begin{aligned} S(\Theta^2) = &-\mu_3 \int d^4\zeta e^{-\phi}\sqrt{\det g}\,\Theta^T\Gamma^0\left(\frac{1}{48}\Gamma^{mnp}H_{mnp} - \frac{1}{16}\Gamma^{\tilde{i}pq}H_{\tilde{i}pq}\right)\Theta + \\ &+\frac{\mu_3}{32}\int d^4\zeta\sqrt{\det g}\,\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Theta^T\Gamma^0\left(\frac{1}{36}\Gamma_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}{}^{pqr}F'_{pqr} + \Gamma_{\tilde{i}\tilde{j}}{}^{p}F'_{\tilde{k}\tilde{l}p}\right)\Theta\,. \end{aligned} \tag{A.41}$$

Here note that the first term in the second line vanishes for the case when the flux is turned on only along the compact directions. As a result we get

$$S(\Theta^2) = \frac{\mu_3}{16}\int d^4\zeta\sqrt{\det g}\,\Theta^T\Gamma^0\left(e^{-\phi}\left\{\Gamma^{\tilde{i}pq}H_{\tilde{i}pq} - \frac{1}{3}\Gamma^{mnp}H_{mnp}\right\} + \frac{1}{2}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Gamma_{\tilde{i}\tilde{j}}{}^{p}F'_{\tilde{k}\tilde{l}p}\right)\Theta\,. \tag{A.42}$$

In the following we will first consider the case when the three brane wraps the divisor $Z^3 =$ constant and concentrate ourselves to the choice of flux as given by eq. (5.3). The above action can be rewritten as

$$S(\Theta^2) = \frac{\mu_3}{16}\int d^4\zeta\sqrt{\det g}\,\Theta^T\Gamma^0 M\Theta\,, \tag{A.43}$$

with the matrix M defined to be

$$M = \left(e^{-\phi}\Gamma^{\tilde{i}\tilde{j}p}H_{\tilde{i}\tilde{j}p} + \frac{1}{2}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Gamma_{\tilde{i}\tilde{j}}{}^{p}F'_{\tilde{k}\tilde{l}p}\right). \tag{A.44}$$

The index p now take value only along directions orthogonal to the divisor.

It is convenient to choose a basis, where the components of Θ are labelled as $|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle$, where $\epsilon_j = \pm 1$, $j = 1, \ldots, 3$ refer to ($-i$ times) the eigen values of $\Gamma^{\hat{x}^j\hat{y}^j}$ respectively.[11] The label $a = \pm 1$ refers to the SO(4) subgroup of the rotation group SO(10). Before proceeding, let us note here that from the commutation relations for the Γ matrices it follows that $\Gamma^{\hat{y}^1\hat{y}^2\hat{y}^3}$ squares to -1 and as a result, $\Gamma^{\hat{y}^1\hat{y}^2\hat{y}^3}|\epsilon_1, \epsilon_2, \epsilon_3, a\rangle$ can never vanish.

We will now evaluate the matrix, M, eq. (A.44), in this basis. We start with the first term, $e^{-\phi}\Gamma^{ijp}H_{ijp}$ which arises from the DBI term. From eq. (5.3) it is easy to see that it takes the form,

$$\begin{aligned} M_{DBI} = e^{-\phi}\Big[&\Gamma^{x^1x^2x^3} - 2\Gamma^{y^1y^2y^3} - (\Gamma^{x^2x^3y^1} + \Gamma^{x^3x^1y^2} + \Gamma^{x^1x^2y^3}) + \\ &+ (\Gamma^{y^2y^3x^1} + \Gamma^{y^3y^1x^2} + \Gamma^{y^1y^2x^3})\Big]\,. \end{aligned} \tag{A.45}$$

Here we note that the indices refer to the coordinate basis, which is different from the vierbein basis.

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[11] Here and in the following, the repeated index j in $\Gamma^{\hat{x}^j\hat{y}^j}$ as well as in $\left(\Gamma^{x^j}\Gamma^{y^j}\right)$ does not indicate a summation over j.

The metric, with r_a in eq. (5.6) set to unity takes the form

$$ds^2 = \sum_i |dx^i + \tau dy^i|^2 \tag{A.46}$$

where $\tau = e^{\frac{2\pi i}{3}}$. A convenient choice of vierbeins is then

$$e^{\hat{x}^1}_{x^1} = 1\,, \qquad e^{\hat{y}^1}_{x^1} = 0\,, \qquad e^{\hat{x}^1}_{y^1} = \cos\left(\frac{2\pi}{3}\right), \qquad e^{\hat{y}^1}_{y^1} = \sin\left(\frac{2\pi}{3}\right). \tag{A.47}$$

The Γ matrices in the vierbein basis and the coordinate basis are related to each other by

$$\begin{aligned}
\Gamma^{x^i} &= \Gamma^{\hat{x}^i} - \cot\left(\frac{2\pi}{3}\right)\Gamma^{\hat{y}^i}\,, \qquad \Gamma^{y^i} = \operatorname{cosec}\left(\frac{2\pi}{3}\right)\Gamma^{\hat{y}^i}\,,\\
\Gamma_{x^i} &= \Gamma^{\hat{x}^i}\,, \qquad \Gamma_{y^i} = \cos\left(\frac{2\pi}{3}\right)\Gamma^{\hat{x}^i} + \sin\left(\frac{2\pi}{3}\right)\Gamma^{\hat{y}^i}\,.
\end{aligned} \tag{A.48}$$

In particular one finds that $(\Gamma^{y^i})^2 = \operatorname{cosec}^2(\frac{2\pi}{3}) = \frac{4}{3}$. After some more algebra we can then write M_{DBI} as,

$$\begin{aligned}
M_{DBI} &= e^{-\phi}\left\{\left(\frac{3}{4}\right)^3 \Gamma^{x^1}\Gamma^{y^1}\Gamma^{x^2}\Gamma^{y^2}\Gamma^{x^3}\Gamma^{y^3} - \right.\\
&\quad - \left(\frac{3}{4}\right)^2 \left[\Gamma^{x^2}\Gamma^{y^2}\Gamma^{x^3}\Gamma^{y^3} + \Gamma^{x^1}\Gamma^{y^1}\Gamma^{x^2}\Gamma^{y^2}\Gamma^{x^1}\Gamma^{y^1}\Gamma^{x^3}\Gamma^{y^3}\right] + \\
&\quad \left. + \frac{3}{4}\left[\Gamma^{x^1}\Gamma^{y^1} + \Gamma^{x^2}\Gamma^{y^2} + \Gamma^{x^3}\Gamma^{y^3}\right] - 2\right\}\Gamma^{y^1}\Gamma^{y^2}\Gamma^{y^3}\,.
\end{aligned} \tag{A.49}$$

From eq. (A.48) we get that

$$\begin{aligned}
\left(\Gamma^{x^i}\Gamma^{y^i}\right)|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle &= \operatorname{cosec}\left(\frac{2\pi}{3}\right)\left(\Gamma^{\hat{x}^i\hat{y}^i} - \cot\left(\frac{2\pi}{3}\right)\right)|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle\\
&= \operatorname{cosec}^2\left(\frac{2\pi}{3}\right) e^{\frac{i\pi}{3}\epsilon_i}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle\,.
\end{aligned} \tag{A.50}$$

It then follows that M_{DBI} acting on the state $|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle$ is

$$M_{DBI}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle = \left(\frac{4}{3}\right)\mathcal{M}\;\Gamma^{\hat{y}^1\hat{y}^2\hat{y}^3}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle \tag{A.51}$$

where,

$$\begin{aligned}
\mathcal{M} = &\left\{e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2+\epsilon_3)} - 2 - e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2)} - e^{-\frac{i\pi}{3}(\epsilon_2+\epsilon_3)} - e^{-\frac{i\pi}{3}(\epsilon_3+\epsilon_1)} + \right.\\
&\left. + e^{-\frac{i\pi}{3}\epsilon_1} + e^{-\frac{i\pi}{3}\epsilon_2} + e^{-\frac{i\pi}{3}\epsilon_3}\right\}.
\end{aligned} \tag{A.52}$$

Similarly we can evaluate the second term in eq. (A.44) which arises due to the WZ terms,

$$M_{WZ} = \frac{1}{2}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Gamma^{p}_{\tilde{i}\tilde{j}}(F_{\tilde{k}\tilde{l}p} - CH_{\tilde{k}\tilde{l}p})\,. \tag{A.53}$$

Here p takes values only over directions orthogonal to the divisor. It is easy to see that with an appropriate choice of orientation for the divisor,[12]

$$\frac{1}{2}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}}\Gamma_{\tilde{i}\tilde{j}}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle = \epsilon_1\epsilon_2\Gamma^{\tilde{k}\tilde{l}}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle\,. \tag{A.54}$$

Thus,

$$M_{WZ}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle = \epsilon_1\epsilon_2\Gamma^{\tilde{k}\tilde{l}p}(F_{\tilde{k}\tilde{l}p} - CH_{\tilde{k}\tilde{l}p})|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle\,. \tag{A.55}$$

A little more algebra then shows that this can be written as,

$$M_{WZ}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle = \frac{4}{3\sqrt{3}}\epsilon_1\epsilon_2\{\mathcal{M} + 1 + e^{-\frac{i\pi}{3}(\epsilon_1+\epsilon_2+\epsilon_3)}\}\Gamma^{\hat{y}^1\hat{y}^2\hat{y}^3}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle \tag{A.56}$$

Adding, eq. (A.51), eq. (A.56) we finally get that M acting on $|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle$ is given by, eq. (5.9).

As discussed above $\Gamma^{\hat{y}^1\hat{y}^2\hat{y}^3}|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle$ cannot vanish. Thus the zero modes of M can only arise if m, eq. (5.10) vanishes. A quick inspection shows that this happens when $(\epsilon_1,\epsilon_2,\epsilon_3)$ all becomes either 1 or -1. Thus these values of $(\epsilon_1,\epsilon_2,\epsilon_3)$ give rise to zero modes. Also m does not vanish for any other choice of $(\epsilon_1,\epsilon_2,\epsilon_3)$. Therefore there are no other zero modes. Finally, one also finds that the two states $|\epsilon_1,\epsilon_2,\epsilon_3,a\rangle$ and $|\epsilon_1,\epsilon_2,-\epsilon_3,a\rangle$, which are related by U(1) rotations in the $x^3 - y^3$ plane, have a different mass in general. This is to be expected since the U(1) symmetry is broken by the flux as discussed in section 5.

Let us also now briefly discuss the case of the more general divisor $n_iZ^i = c$. By relabelling the Z^i coordinates if necessary we can always take $n^3 \neq 0$. In this case it is useful to choose coordinates, ψ^1,ψ^2,ψ^3 which are related to the coordinates Z^i as follows:

$$\begin{aligned} Z^1 &= n_1\psi^3 + n_3\psi^1 \\ Z^2 &= n_2\psi^3 + n_3\psi^2 \\ Z^3 &= -n_1\psi^1 - n_2\psi^2 + n_3\psi^3\,. \end{aligned} \tag{A.57}$$

ψ^1,ψ^2 are parallel to the divisor and ψ^3 is orthogonal to it. The divisor in these coordinates can be written as $\psi^3 = \mathit{constant}$. The flux $G = F - \Phi H$ can be expressed as

$$G = \mathit{const}(d\psi^1\wedge d\psi^2\wedge d\bar{\psi}^3 + d\psi^2\wedge d\psi^3\wedge d\bar{\psi}^1 + d\psi^3\wedge d\psi^1\wedge d\bar{\psi}^2)\,. \tag{A.58}$$

Upto a constant this is exactly the form of G in the Z^i coordinates. A further change of variables,

$$\begin{aligned} \tilde{\psi}^1 &= \sqrt{n_3^2+n_1^2}\psi^1 + \sqrt{n_3^2+n_2^2}\psi^2 \\ \tilde{\psi}^2 &= \sqrt{n_3^2+n_1^2}\psi^1 - \sqrt{n_3^2+n_2^2}\psi^2 \\ \tilde{\psi}^3 &= \psi^3\,, \end{aligned} \tag{A.59}$$

[12] An opposite choice of orientation corresponds to a negative sign on the r.h.s below. This still gives the same number of zero modes.

preserves the form of G, eq. (A.58). It also allows the metric to be written in diagonal form as,

$$ds^2 = \sum_i r_i^2 |d\tilde{\psi}^i|^2 \,. \tag{A.60}$$

This is the same as the metric in the Z^i coordinates we considered eq. (5.6). Thus the analysis for the general divisor maps after a change of coordinates to the case $Z^3 = c$. And we learn that for a general divisor also there are four fermion zero modes.

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